



On Optimal Personal Income Taxation

Paweł Doligalski

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

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I confirm that chapter *Optimal Redistribution with a Shadow Economy* was jointly co-authored with Luis Rojas and I contributed 60% of the work.

Paweł Doligalski

A handwritten signature in black ink that reads 'Paweł Doligalski'.

15/06/2016, Florence

Babci / To my Grandma

Acknowledgements

“I BLAME ALL of you. Writing this book has been an exercise in sustained suffering. The casual reader may, perhaps, exempt herself from excessive guilt, but for those of you who have played the larger role in prolonging my agonies with your encouragement and support, well... you know who you are, and you owe me.”

(Brendan Pietsch, “Dispensational Modernism”)

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Thesis summary

How should we tax people's incomes? I address this question from three different angles. The first chapter describes the optimal income tax when people can hide earnings by working in a shadow economy. The second chapter examines the optimal taxation of employees when firms can insure their workers and help them avoid taxes. The final chapter shows that a basic income policy - an unconditional cash transfer to every citizen - can, under certain conditions, be justified on efficiency grounds.

In 'Optimal Redistribution with a Shadow Economy', written jointly with Luis Rojas, we examine the constrained efficient allocations in the [Mirrlees \(1971\)](#) model with an informal sector. There are two labor markets: formal and informal. The planner observes only income from the formal market. We show that the shadow economy can be welfare improving through two channels. It can be used as a *shelter against tax distortions*, raising the efficiency of labor supply, and as a *screening device*, benefiting redistribution. We calibrate the model to Colombia, where 58% of workers are employed informally. The optimal share of shadow workers is close to 22% for the Rawlsian planner and less than 1% for the Utilitarian planner. Furthermore, we find that the optimal tax schedule is very different than the one implied by the [Mirrlees \(1971\)](#) model without the informal sector.

New Dynamic Public Finance describes the optimal income tax in the economy without private insurance opportunities. In 'Optimal Taxation with Permanent Employment Contracts' I extend this framework by introducing permanent employment contracts which facilitate insurance provision within firms. The optimal tax system becomes remarkably simple, as the government outsources most of the insurance provision to employers and focuses mainly on redistribution. When the government wants to redistribute to the poor, a dual labor market can be optimal. Less productive workers are hired on a fixed-term basis and are partially insured by the government, while the more productive ones enjoy the full insurance provided by the permanent employment. Such arrangement can be preferred, as it minimizes the tax avoidance of top earners. I provide empirical evidence consistent with the theory and characterize the constrained efficient allocations for Italy.

When does paying a strictly positive compensation in every state of the world improve incentives to exert effort? In 'Minimal Compensation and Incentives for Effort' I show that in the typical model of moral hazard it happens only when the effort is a strict complement to consumption. If the cost of effort is monetary, a positive minimal compensation strengthens incentives only when

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the agent is prudent and always does so when the marginal utility of consumption is unbounded at zero consumption. I discuss potential applications of these results in personal income taxation. The minimal compensation can be interpreted as a basic income - an unconditional cash transfer to every citizen. Therefore, I provide an efficiency rationale for the basic income.

1 Optimal Redistribution with a Shadow Economy

Joint with Luis Rojas (European University Institute and Banco de la República)

Abstract

We examine the constrained efficient allocations in the [Mirrlees \(1971\)](#) model with a shadow economy. There are two labor markets: formal and informal. The income from the formal market is observed by the planner, while the income from the informal market is not. There is a distribution of workers that differ with respect to the formal and the informal productivity. We show that when the planner does not observe individual productivities some workers may optimally work in the shadow economy. Moreover, the social welfare of the model with the shadow economy can be higher than the welfare of the model without the informal sector. These results hold even when each agent is more productive formally than in the shadow economy. The model is calibrated to Colombia, where 58% of workers are employed informally. We derive workers' productivities in the two sectors from a household survey. The optimal share of shadow workers is close to 22% for the Rawlsian planner and less than 1% for the Utilitarian planner. The optimal income tax schedule is very different then the one implied by the [Mirrlees \(1971\)](#) model without the informal sector.

1.1 Introduction

Informal activity, defined broadly as any endeavor which is not necessarily illegal but evades taxation, accounts for a large fraction of economic activity in both developing and developed economies.

The views expressed in this article are those of the authors and do not necessarily reflect the position of Banco de la República. We are grateful for useful comments of Árpád Ábrahám, Charles Brendon, Antoine Camous, Hal Cole, Piero Gottardi, Ramon Marimon, Wojciech Kopczuk, Dirk Krueger, Humberto Moreira, Erwin Ooghe, Wojciech Paczos and Evi Pappa, as well as the seminar participants at the WIEM 2014 Conference, the Central Bank of Hungary, the Royal Economic Society 2015 Annual Conference, the IEB Workshop on Economics of Taxation, the Econometric Society 2015 World Congress, the European Economic Association 2015 Annual Congress, the University of Essex, the Bristol University, the University of Mannheim and the University of Barcelona. Paweł Doligalski thanks the Central Bank of Hungary for the possibility of working on this project during his stay there. All mistakes are ours.

According to [Jutting, Laiglesia, et al. \(2009\)](#) more than half of the jobs in the non-agricultural sector worldwide can be considered informal. [Schneider, Buehn, and Montenegro \(2011\)](#) estimate the share of informal production in the GDP of high income OECD countries in the years 1999-2007 as 13.5%. Given this evidence, the informal sector should be considered in the design of fiscal policy. This paper extends the theory of the optimal redistributive taxation by [Mirrlees \(1971\)](#) to the economies with an informal labor market.

The ability of the state to redistribute income depends on how responsive to taxes individuals are. When incomes are very elastic, differential taxation of different individuals is hard, because workers adjust their earnings to minimize the tax burden.¹ The shadow economy allows workers to earn additional income which is unobserved by the government. Without shadow economy, workers respond to taxes only by changing their total labor supply. With the shadow economy, they can additionally shift labor between the formal and the informal sector, which increases the elasticity of their formal income. As incomes in the formal economy become more elastic, redistribution becomes more difficult.

We show that the government can exploit differences in informal productivity between workers to improve redistribution. Suppose there are two types of workers: skilled and unskilled. The responsiveness of the skilled workers determines the taxes they pay and the transfers the unskilled receive. In the world without the shadow economy, this responsiveness depends on how easy it is for the skilled to reduce income to the level of the unskilled worker. If that happens, the government cannot tax differentially the two types of individuals. In the world with the shadow economy, the government can improve redistribution in the following way. By increasing taxes at low levels of formal income, the unskilled workers are pushed to informality. If the unskilled workers can easily find a good informal job, this transition will not hurt them much. Now the skilled workers can avoid taxes only if they too move to the shadow economy. Hence, the responsiveness of the skilled workers depends on their informal productivity. If the skilled workers suffer a large productivity loss by moving to the other sector, the government can tax them more in the formal sector and provide higher transfers to the unskilled informal workers. In the opposite case, however, when the skilled can easily move between sectors while the unskilled cannot, the government cannot use the shadow economy to discourage the skilled workers from reducing formal income. In such a case, redistribution will be reduced.

The shadow economy also affects the efficiency of labor allocation by sheltering workers from tax distortions.² The labor supply of formal workers is determined jointly by their formal productivity and a marginal tax rate they face. In contrast, the labor supply of informal workers depends only

¹[Diamond \(1998\)](#) and [Saez \(2001\)](#) expressed the optimal tax rates in the Mirrlees model with elasticities. The higher is the elasticity of labor supply, the lower is the optimal marginal tax rate at this level of income.

²This effect corresponds to what [La Porta and Shleifer \(2008\)](#) call the romantic view on the shadow economy. In this view, associated with the works of Hernando de Soto ([de Soto \(1990, 2000\)](#)), the informal sector protects productive firms from harmful regulation and taxes.

on their informal production opportunity and is unaffected by tax distortions. When their informal productivity is not much lower than the formal one, informal workers will produce more than if they stayed in the formal sector. In this way the shadow economy improves the allocation of labor and raises efficiency.

Whether the shadow economy is harmful or beneficial from the social welfare perspective depends on its joint impact on redistribution and efficiency. The informal sector improves redistribution if the workers that pay high taxes cannot easily move to the shadow economy. It benefits efficiency if informal workers have similar productivities in formal and informal sector. As a rule of a thumb, we can say that the shadow economy raises welfare if it allows poor workers who collect transfers to earn some additional money, but does not tempt the rich taxpayers to reduce their formal income.

We derive the formula for the optimal tax with a shadow economy. The informal sector imposes an upper bound on the marginal tax rate, which depends on the distribution of formal and informal productivities. The optimal tax rate at each formal income level is given by either the usual [Diamond \(1998\)](#) formula or the upper bound, if the Diamond formula prescribes rates that are too high. In contrast to the standard [Mirrlees \(1971\)](#) model, in the model with shadow economy different types of workers are likely to be bunched at the single level of formal income. Specifically, all agents that supply shadow labor are subject to bunching. We develop the optimal bunching condition which complements the Diamond formula.³

The model is calibrated to Colombia, where 58% of workers are employed informally. We derive the joint distribution of formal and shadow productivity from a household survey. The main difficulty is that most individuals work only in one sector at a time. We infer their productivity in the other sector by estimating a factor: a linear combination of workers' and jobs' characteristics that explains most of the variability of shadow and formal productivities. The factor allows us to match similar individuals and infer their missing productivities. When we apply the actual tax schedule to the calibrated economy, the model replicates well the actual size of the informal sector.

We find that the optimal share of shadow workers in the total workforce is close to 22% under the Rawlsian planner and less than 1% under the Utilitarian planner. This means that the optimal shadow economy is much smaller than the 58%, the actual share of shadow workers in Colombia. In comparison the Colombian income tax at the time, the optimal tax schedule has lower marginal rates at the bottom and higher rates elsewhere. Lower tax rates at the bottom displace less workers to the shadow economy, while higher tax rates above raise more revenue from high earners, yielding large welfare gains. The optimal tax rates are generally lower than the ones implied by the [Mirrlees \(1971\)](#) model without the informal sector. The application of the [Mirrlees \(1971\)](#) income tax would

³In the [Mirrlees \(1971\)](#) model without wealth effects the optimal allocation is described by the Diamond formula if and only if the resulting income schedule is non-decreasing, which is usually verified ex post. If the Diamond formula implies the income schedule that is decreasing at some type, our optimal bunching condition recovers the optimum.

displace an excessive number of workers to the shadow economy.

Related literature. Tax evasion has been studied at least since [Allingham and Sandmo \(1972\)](#). For us, the most relevant paper from this literature is [Kopczuk \(2001\)](#). He shows that tax evasion can be welfare improving if and only if individuals are heterogeneous with respect to both productivity and tax evasion ability.⁴ We explore this result by decomposing the welfare gain from tax evasion into the efficiency and redistribution components. Furthermore, [Kopczuk \(2001\)](#) derives the optimal linear income tax with tax evasion. We focus on the optimal non-linear income tax and provide a sharp characterization of the optimal shadow economy. [Frías, Kumler, and Verhoogen \(2013\)](#) show that underreporting of wages decreases, once reported income is linked to pension benefits. [Waseem \(2013\)](#) documents that an increase of taxes of partnerships in Pakistan led to a massive shift to other business forms as well as a large spike in income underreporting.

Our model is focused on the workers' heterogeneity with respect to formal and informal productivities. A similar approach was taken by [Albrecht, Navarro, and Vroman \(2009\)](#), who study the impact of labor market institutions in a model with the formal and informal labor markets and a search friction. There is a complementary approach to modeling the shadow economy, which focuses on firms' rather than workers' heterogeneity. In [Rauch \(1991\)](#) managers with varying skills decide in which sector to open a business. He finds that less productive managers choose informal sector in order to avoid costly regulation. [Meghir, Narita, and Robin \(2015\)](#) consider heterogeneous firms that decide in which sector to operate and who are randomly matched with homogeneous workers. They find that policies aimed at reduction of the shadow economy increase competition for workers in the formal labor market and improve welfare. [Amaral and Quintin \(2006\)](#) to the best of our knowledge provide the only framework with the shadow economy where heterogeneity of both firms and workers is present. They extend the [Rauch \(1991\)](#) model by allowing for physical and human capital accumulation. Due to complementarity between the two types of capital, educated workers tend to stay in the more capital intensive formal sector.

The following two papers derive the optimal policy in related environments. [Gomes, Lozachmeur, and Pavan \(2014\)](#) study the optimal sector-specific income taxation when individuals can work in one of the two sectors of the economy. In our setting there are also two sectors, but the government can impose tax only on one of them. Moreover, we allow agents to work in the two sectors simultaneously. [Alvarez-Parra and Sánchez \(2009\)](#) study the optimal unemployment insurance with the moral hazard in search effort and an informal labor market. It is another environment with information frictions in which the informal employment is utilized in the optimal allocation.

⁴[Kopczuk \(2001\)](#) describes his framework as a model of tax avoidance. In our view his results are applicable also in studying tax evasion, which is the focus of our paper.

Structure of the paper. In the next section we use a simple model of two types to show how the shadow economy can emerge in the optimum and what are the welfare consequences. In Section 1.3 we derive the optimal tax schedule with a large number of types and general social preferences. In Section 1.4 we introduce our methodology of extracting shadow productivities from the micro data and apply it to Colombia. We derive the optimal Colombian tax schedule in Section 1.5. The last section concludes.

1.2 Simple model

Imagine an economy inhabited by people that share preferences but differ in productivity. There are two types of individuals, indexed by letters L and H , with strictly positive population shares μ_L and μ_H . They all care about consumption c and labor supply n according to the utility function

$$U(c, n) = c - v(n). \quad (1.1)$$

We assume that v is increasing, strictly convex, twice differentiable and satisfies $v'(0) = 0$. The inverse function of v' is denoted by g .

There are two labor markets and, correspondingly, each agent is equipped with two linear production technologies. An agent of type $i \in \{L, H\}$ produces with productivity w_i^f in a formal labor market, and with productivity w_i^s in an informal labor market. Type H is more productive in the formal market than type L : $w_H^f > w_L^f$. Moreover, in this section we assume that each type's informal productivity is lower than formal productivity: $\forall_i w_i^f > w_i^s$. We relax this assumption when we consider the full model.

Any agent may work formally, informally, or in both markets simultaneously. An agent of type i works n_i hours in total, which is the sum of n_i^f hours at the formal job and n_i^s hours in the shadow economy. The formal and the informal income, denoted by y_i^f and y_i^s respectively, is a product of the relevant productivity and the relevant labor supply. The allocation of resources may involve transfers across types, so one's consumption may be different than the sum of formal and informal income. In order to capture these flows of resources, we introduce a tax T_i , equal to the gap between total income and consumption

$$T_i \equiv y_i^f + y_i^s - c_i. \quad (1.2)$$

A negative tax is called a transfer, and we are going to use these terms interchangeably.

The social planner follows John Rawls' theory of justice and wants to improve the well-being of the least well-off agents,⁵ but is limited by imperfect knowledge. The planner knows the structure and

⁵We pick this particular point of the Pareto frontier because it allows us to show the interesting features of the model with relatively easy derivations. At the end of this section we discuss how other constrained efficient allocations look like.

parameters of the economy, but, as in the standard Mirrlees model, does not observe the type of any individual. In addition, shadow income and labor are unobserved by the planner as well. The only variables at the individual level the planner sees and can directly verify are the formal income y_i^f and the tax T_i . We can think about y_i^f and $y_i^f - T_i$ as a pre-tax and an after-tax reported income. Although shadow labor cannot be controlled directly, it is influenced by the choice of formal labor. Formal labor affects the marginal disutility from labor and hence changes the agent's optimal choice of shadow hours. Two types of labor are related according to the following function, implied by the agent's first order condition

$$n_i^s(n^f) = \max \left\{ g(w_i^s) - n^f, 0 \right\}. \quad (1.3)$$

When the agent works a sufficient number of hours in the formal sector, the marginal disutility from labor is too high to work additionally in the shadows. However, if the formal hours fall short of $g(w_i^s)$, the resulting gap is filled with shadow labor.

The planner maximizes the Rawlsian social welfare function, given by a utility level of the worst-off agent

$$\max_{\left\{ (n_i^f, T_i) \in \mathbb{R}_+ \times \mathbb{R} \right\}_{i \in \{L, H\}}} \min \{ U(c_L, n_L), U(c_H, n_H) \}, \quad (1.4)$$

subject to the relation between formal and shadow labor

$$n_i^s(n^f) = \max \left\{ g(w_i^s) - n^f, 0 \right\}, \quad (1.5)$$

the accounting equations

$$\forall_{i \in \{L, H\}} c_i = w_i^f n_i^f + w_i^s n_i^s(n_i^f) - T_i, \quad (1.6)$$

$$\forall_{i \in \{L, H\}} n_i = n_i^f + n_i^s(n_i^f), \quad (1.7)$$

a resource constraint

$$\sum_{i \in \{L, H\}} \mu_i T_i \geq 0, \quad (1.8)$$

and incentive-compatibility constraints

$$\forall_{i \in \{L, H\}} U(c_i, n_i) \geq U \left(w_{-i}^f n_{-i}^f + w_i^s n_i^s \left(\frac{w_{-i}^f}{w_i^f} n_{-i}^f \right) - T_{-i}, \frac{w_{-i}^f}{w_i^f} n_{-i}^f + n_i^s \left(\frac{w_{-i}^f}{w_i^f} n_{-i}^f \right) \right). \quad (1.9)$$

We denote the generic incentive constraint by $IC_{i,-i}$. It means that an agent i cannot be better off by earning the formal income of the other type and simultaneously adjusting informal labor.

1.2.1 First-best

What if the planner is omniscient and directly observes all variables? The planner knows types and can choose the shadow labor supply directly. The optimal allocation is a solution to the welfare maximization problem (1.4) where planner chooses both formal and shadow labor and a tax of each type subject only to the accounting equations (1.6) and (1.7) and the resource constraint (1.8). All types are more productive in the formal sector than in the shadow economy, so no agent will work informally. Each agent will supply the formal labor efficiently, equalizing the marginal social cost and benefit of working. Moreover, the planner redistributes income from H to L in order to achieve the equality of well-being.

Proposition 1.1. *In the first-best both types work only formally and supply an efficient amount of labor: $\forall_i v'(n_i) = w_i^f$. Utility levels of the two types are equal: $U(c_L, n_L) = U(c_H, n_H)$.*

We can slightly restrict the amount of information available to the planner without affecting the optimal allocation. Suppose that the planner still observes the formal productivity, but shadow labor and income are hidden. The optimal allocation is a solution to (1.4) subject to the relation between shadow and formal labor (1.5), the accounting equations (1.6) and (1.7) and the resource constraint (1.8).

Proposition 1.2. *If the planner knows types, but does not observe shadow labor and income, the planner can achieve the first-best.*

When the types are known, the planner can use the lump-sum taxation and implement the first-best. Without additional frictions, the hidden shadow economy does not constrain the social planner.

1.2.2 Second-best

Let's consider the problem in which neither type nor informal activity is observed. The planner solves (1.4) subject to all the constraints (1.5) - (1.9). We call the solution to this problem the second-best or simply the optimum.

Proposition 1.3. *The optimum is not the first-best. $IC_{H,L}$ is binding, while $IC_{L,H}$ is slack.*

In the first-best, both types work only on the formal market and their utilities are equal. If H could mimic the other type, higher formal productivity would allow H to increase utility. Hence, the first-best does not satisfy $IC_{H,L}$ and this constraint limits the welfare at the optimum. On the other hand, $IC_{L,H}$ never binds at the optimum. It would require the redistribution of resources from type L to H , which is clearly suboptimal.

Optimal shadow economy

The standard Mirrlees model typically involves labor distortions, since they can relax the binding incentive constraints. If type i is tempted to pretend to be of the type $-i$, distorting number of hours of $-i$ will discourage the deviation. Agents differ in labor productivity, so if i is more (less) productive than the other type, decreasing (increasing) number of hours worked by $-i$ will make the deviation less attractive. Proposition 1.3 tells us that no agent wants to mimic type H , hence the planner has no reason to distort the labor choice of these agents. Moreover, according to (1.5) shadow labor is supplied only if formal labor is sufficiently distorted. Hence, the classic result of no distortions at the top implies here that H will work only formally.

Corollary 1. *Type H faces no distortions and never works in the shadow economy.*

On the other hand, the planner can improve social welfare by distorting the formal labor supply of type L . Stronger distortions relax the binding incentive constraint and allow the planner to redistribute more. If distortions are strong enough, type L will end up supplying shadow labor. Optimality of doing so depends on whether and by how much increasing shadow labor of type L relaxes the binding incentive constraint. As Proposition 1.4 demonstrates, a comparative advantage of type L in shadow labor plays a crucial role. In the proof we use the optimality condition derived in the appendix (see Lemma 1.3). In order to make sure that this condition is well behaved, we require that v'' is nondecreasing.⁶

Proposition 1.4. *Suppose that v'' is nondecreasing. Type L may optimally work in the shadow economy only if*

$$\left(\frac{w_L^s}{w_L^f} - \frac{w_H^s}{w_H^f} \right) \mu_H \geq \frac{w_L^f - w_L^s}{w_L^f} \mu_L. \quad (1.10)$$

Condition (1.10) is also a sufficient condition for type L to optimally work in the shadow economy if $\frac{w_H^f}{w_L^f} g(w_H^s) \geq g(w_L^s)$. Otherwise, the sufficient (but not necessary) condition is

$$\left(\frac{w_L^s}{w_L^f} - \frac{v' \left(\frac{w_L^f}{w_H^f} g(w_L^s) \right)}{w_H^f} \right) \mu_H \geq \frac{w_L^f - w_L^s}{w_L^f} \mu_L. \quad (1.11)$$

Inequality (1.10) provides a necessary condition for the optimal shadow economy by comparing the marginal benefit and cost of increasing shadow labor of type L . The left hand side is the comparative advantage of type L over type H in the shadow labor, multiplied by the share of type H . This advantage has to be positive for type L to optimally work in the shadow economy.

⁶In the canonical case of isoelastic utility, it means that the elasticity of the labor supply is not greater than 1.

Otherwise, increasing shadow labor of this type does not relax the binding incentive constraint. Since the shadow economy does not facilitate screening of types, there are no benefits from the productivity-inferior shadow sector. The welfare gains from the relaxed incentive constraint are proportional to the share of type h , as the planner obtains more resources for redistribution by imposing a higher tax on this type. On the right hand side, the cost of increasing shadow labor is given by the productivity loss from using the inferior shadow production, multiplied by the share of types that supply shadow labor.

Condition (1.10) is also a sufficient condition for type L to work in the shadow economy if the shadow productivity of type H is not much lower than the shadow productivity of type l . If that is not the case, the optimality condition derived in Lemma 1.3 is not sufficient and we have to impose a stronger sufficiency condition (1.11).

Figure 1.1 illustrates the proposition on the diagram of the parameter space (w_H^s, w_L^s) . Along the diagonal no type has the comparative advantage, since ratios of shadow and formal productivity of the two types are equal. The optimal shadow economy requires that type L has the comparative advantage in shadow labor, so the interesting action happens above the diagonal. The shadow economy is never optimal for pairs of shadow productivities which violate inequality (1.10). Depending on whether $\frac{w_H^f}{w_L^f} g(w_H^s)$ is greater than $g(w_L^s)$, the inequality (1.10) is also a sufficient condition for the optimal shadow economy, or we use (1.11) instead. Note that the lower frontier of the necessity region crosses the vertical axis at the value $\mu_L w_L^f$. As the proportion of type L decreases toward zero, the region where shadow economy is optimal increases, in the limit encompassing all the points where type L has the comparative advantage over H in shadow labor.

We know when type L optimally works in the shadow economy. Proposition 1.5 tells us, how much shadow labor should type L supply in this case.

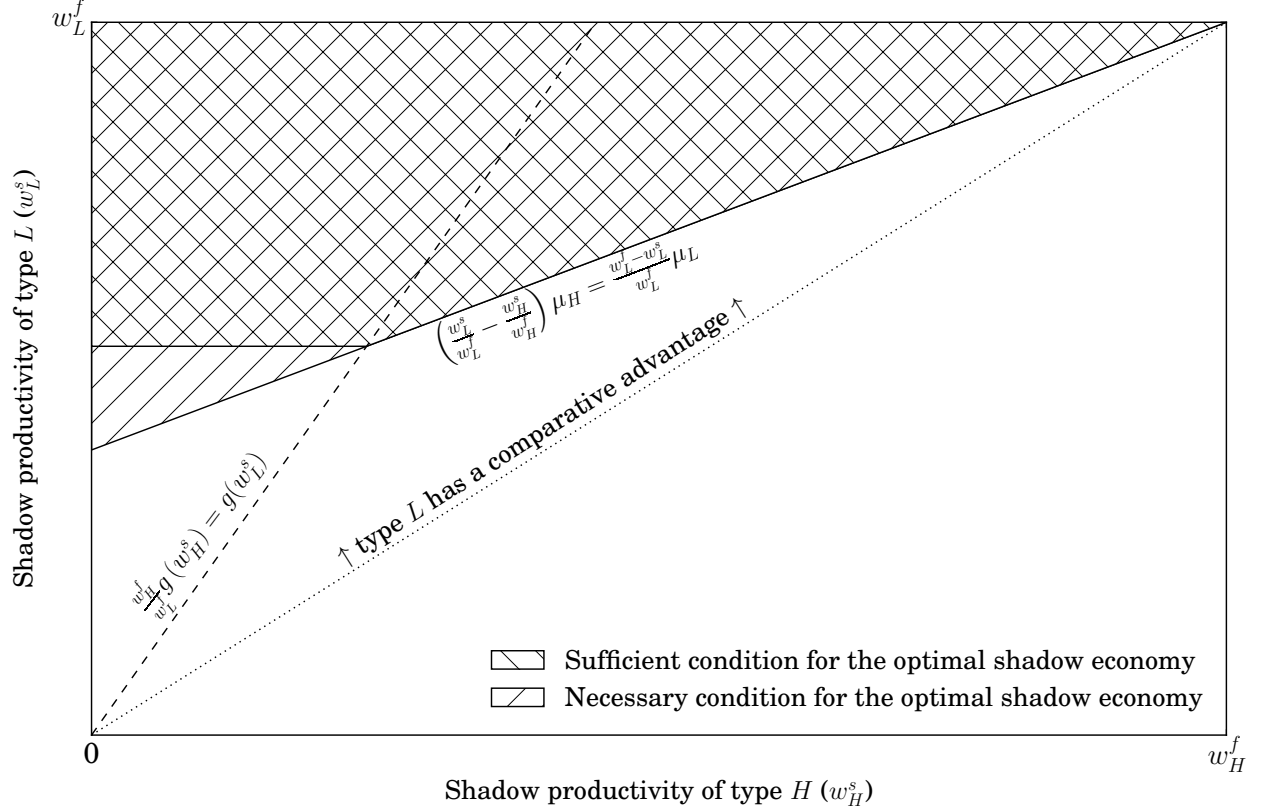
Proposition 1.5. *Suppose that type L optimally works in the shadow economy. Type L works only in the shadow economy if $w_H^s \geq w_L^s$. Type L works in both sectors simultaneously if $w_H^s < w_L^s$.*

When type L is more productive in the shadows than H and works only in the shadow economy, then by $IC_{H,L}$ the utility of type L will be greater than the utility of H . Since the planner is Rawlsian, the utility levels of both types will be equalized by making type L work partly in the formal economy. On the other hand, when type H is more productive informally, $IC_{H,L}$ means that the utility of type L will be always lower. Then if the shadow economy benefits type L , the planner will use it as much as possible.

Shadow economy and welfare

In order to examine the welfare implications of the shadow economy, we compare social welfare of the two allocations. The first one, noted with a superscript M , is the optimum of the standard

Figure 1.1: The optimal shadow economy



Mirrlees model. We can think about the standard Mirrlees model as a special case of our model, in which both w_L^s and w_H^s are equal 0. The second allocation, noted with a superscript SE , involves type L working only in the shadow economy and the planner transferring resources from type H to type L up to the point when the incentive constraint $IC_{H,L}$ binds. The allocation SE is not necessarily the optimum of the shadow economy model. We use it, nevertheless, to illuminate the channels through which the shadow economy influences social welfare. We measure social welfare with the utility of type L . The welfare difference between the two allocations can be decomposed in the following way

$$\underbrace{U(c_L^{SE}, n_L^{SE}) - U(c_L^M, n_L^M)}_{\text{total welfare gain}} = \underbrace{U(w_L^s n_L^{SE}, n_L^{SE}) - U(w_L^f n_L^M, n_L^M)}_{\text{efficiency gain}} + \underbrace{T_L^M - T_L^{SE}}_{\text{redistribution gain}}. \quad (1.12)$$

The efficiency gain measures the difference in distortions imposed on type L , while the redistribution gain describes the change in the level of transfer type L receives. Thanks to the quasilinear preferences, we can decompose these two effects additively.

Efficiency gain. The distortion imposed on type L in the shadow economy arise from the productivity loss $w_L^f - w_L^s$. By varying w_L^s , this distortion can be made arbitrarily small. On the other hand, the distortion of the standard Mirrlees model is implied by the marginal tax rate on formal income. Given redistributive social preferences, it is always optimal to impose a positive tax rate on type l . The efficiency gain, which captures the difference in distortions between two regimes, is strictly increasing in w_L^s . Intuitively, the positive efficiency gain means that the shadow economy raises social welfare by *sheltering the workers from tax distortions*.

Redistribution gain. The shadow economy improves redistribution if the planner is able to give higher transfer to type L (or equivalently raise higher tax from type H). The difference in transfers can be expressed as

$$T_L^M - T_L^{SE} = \mu_H \left(U \left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) - U \left(w_H^s n_H^{SE}, n_H^{SE} \right) \right). \quad (1.13)$$

What determines the magnitude of redistribution is the possibility of production of type H after misreporting. In the standard Mirrlees model deviating type H uses formal productivity and can produce only as much output as type l . In the allocation where type L works only informally, type H cannot supply any formal labor, but is unconstrained in supplying informal labor. Hence, the redistribution gain is strictly decreasing in w_H^s . Intuitively, a positive redistribution gain means that the shadow economy is used as a *screening device*, helping the planner to tell the types apart.

Proposition 1.6 uses the decomposition into the efficiency and redistribution gains in order to derive threshold values for shadow productivity of each type. Depending on which side of the thresholds the productivities are, the existence of the shadow economy improves or deteriorates social welfare in comparison to the standard Mirrlees model.

Proposition 1.6. *Define an increasing function $H(w^s) = U(w^s g(w^s), g(w^s))$ and the following threshold values*

$$\bar{w}_L^s = H^{-1} \left(U \left(w_L^f n_L^M, n_L^M \right) \right) \in (0, w_L^f), \quad \bar{w}_H^s = H^{-1} \left(U \left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M \right) \right) \in (0, w_H^f). \quad (1.14)$$

If $w_L^s \geq \bar{w}_L^s$ and $w_H^s \leq \bar{w}_H^s$, where at least one of these inequalities is strict, the existence of the shadow economy improves welfare in comparison to the standard Mirrlees model.

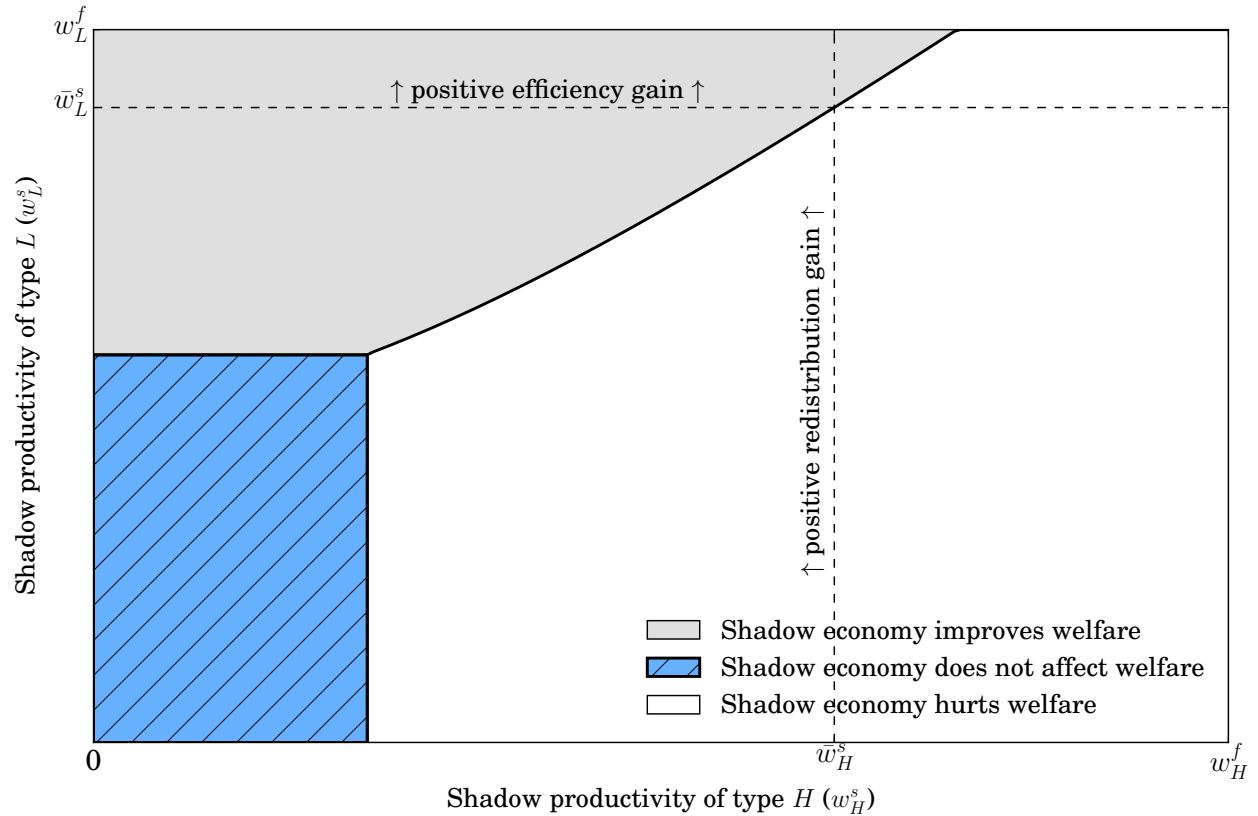
If $w_L^s \leq \bar{w}_L^s$ and $w_H^s \geq \bar{w}_H^s$, where at least one of these inequalities is strict, the existence of the shadow economy deteriorates welfare in comparison to the standard Mirrlees model.

The proposition is illustrated on the Figure 1.2. When the shadow productivity of type L is above \bar{w}_L^s , the efficiency gain is positive. When the shadow productivity of type H is above \bar{w}_H^s , the

OPTIMAL REDISTRIBUTION WITH A SHADOW ECONOMY

redistribution gain is negative. Obviously, when both gains are positive (negative), the shadow economy benefits (hurts) welfare. However, the shadow economy does not have to strengthen both redistribution and efficiency simultaneously to be welfare improving. Particularly interesting is the region where the redistribution gain is negative, but the efficiency gain is sufficiently high such that the welfare is higher with the shadow economy. In this case the optimum of the shadow economy model Pareto dominates the optimum of the Mirrlees model. Type L gains, since the welfare is higher with the shadow economy. Type H benefits as well, as the negative redistribution gain implies a lower tax of this type.

Figure 1.2: Shadow economy and welfare



General social preferences

In this short section we will derive some properties of the whole Pareto frontier of the two-types model. We consider the planner that maximizes the general utilitarian social welfare function

$$\lambda_L \mu_L U(c_L, n_L) + \lambda_H \mu_H U(c_H, n_H), \quad (1.15)$$

where the two Pareto weights are non-negative and sum up to 1. The maximization is subject to the constraints (1.5) - (1.9).

From the Rawlsian case we know that the comparative advantage of type L in shadow labor is necessary for this type to work in the shadows. Proposition 1.7 generalizes this observation.

Proposition 1.7. *Type $i \in \{L, H\}$ may optimally work in the shadow economy only if $\frac{w_i^s}{w_i^f} > \frac{w_{-i}^s}{w_{-i}^f}$ and $\lambda_i > \lambda_{-i}$.*

In order to optimally work in the shadow economy, any type $i \in \{L, H\}$ has to satisfy two requirements. First, type i needs to have the comparative advantage in the shadow labor over the other type. Otherwise, shifting labor from formal to shadow sector does not relax the incentive constraints. Second, the planner has to be willing to redistribute resources to type i - the Pareto weight of this type has to be greater than the weight of the other type. The shadow economy can be beneficial only when it relaxes the binding incentive constraints, and the incentive constraint $IC_{-i,i}$ binds if $\lambda_i > \lambda_{-i}$. Intuitively, if the planner prefers to tax rather than support some agents, it is suboptimal to let them evade taxation.

When will type i optimally work in the shadow economy? Let's compare the welfare of two allocations. In the first allocation (denoted by superscript SE) type i works exclusively in the shadow economy. It provides the lower bound on welfare when type i is employed informally. The second allocation (denoted by M) is the optimum of the standard Mirrlees model, or equivalently the optimum of the shadow economy model where $w_i^s = w_{-i}^s = 0$. It is the upper bound on welfare when type i is employed only in the formal sector. We can decompose the welfare difference between these two allocations in the familiar way

$$\underbrace{W^{SE} - W^M}_{\text{total welfare gain}} = \underbrace{\mu_i \lambda_i \left(U(w_i^s n_i^{SE}, n_i^{SE}) - U(w_i^f n_i^M, n_i^M) \right)}_{\text{efficiency gain}} + \underbrace{\mu_i (\lambda_i - \lambda_{-i}) (T_i^M - T_i^{SE})}_{\text{redistribution gain}}. \quad (1.16)$$

The welfare difference can be decomposed into the difference in effective distortions imposed on type i and the difference in transfers received by this type. The only essential change in comparison to the simpler Rawlsian case given by (1.12) comes from the Pareto weights. The more the planner cares about type $-i$, the less valuable are gains in redistribution in comparison to the gains in efficiency.

Proposition 1.8. *Suppose that $\lambda_i > \lambda_{-i}$ for some $i \in \{L, H\}$. Define the following thresholds*

$$\bar{w}_i^s = H^{-1} \left(U(w_i^f n_i^M, n_i^M) \right) \in (0, w_i^f), \quad \bar{w}_{-i}^s = H^{-1} \left(U(w_i^f n_i^M, \frac{w_i^f}{w_{-i}^f} n_i^M) \right) \in (0, w_{-i}^f). \quad (1.17)$$

If $w_i^s \geq \bar{w}_i^s$ and $w_{-i}^s \leq \bar{w}_{-i}^s$, where at least one of these inequalities is strict, then type i optimally works in the shadow economy and the optimum welfare is strictly higher than in the standard Mirrlees model.

Proposition 1.8 generalizes the thresholds from Proposition 1.6. Interestingly, when the planner cares more about the more productive formally type H , these agents may end up working in the shadow economy. It may be surprising, since in the standard Mirrlees model the formal labor supply of this type is optimally either undistorted, or distorted upwards, while supplying shadow labor requires a downwards distortion. Nevertheless, if shadow economy magnifies productivity differences between types, it may be in the best interest of type H to supply only informal labor and enjoy higher transfer financed by the other type. The shadow economy in such allocation works as a tax haven, accessible only to the privileged.

1.3 Full model

In this section we describe the optimal tax schedule in the economy with a large number of types. Below we introduce a general taxation problem. Then we examine the requirements of incentive compatibility, which will involve the standard monotonicity condition. We proceed to characterize the optimal income tax. First we derive optimality conditions (which we call the *interior optimality conditions*) under the assumption that the monotonicity condition holds. It is a common practice in the literature on Mirrleesian taxation to stop here and verify the monotonicity numerically ex post. It is justified, since in the standard Mirrlees model the violation of the monotonicity requires rather unusual assumptions. On the other hand, the shadow economy provides an environment where the monotonicity condition is much more likely to be violated. We discuss in detail why it is the case and carry on to the optimality conditions when the monotonicity constraint is binding. The optimal allocation in this case involves bunching, i.e. some types are pooled together at the kinks of the tax schedule. We derive the optimal bunching condition with an intuitive variational method.⁷ In the last subsection we summarize the main results from the full model.

1.3.1 The planner's problem

Workers are distributed on the type interval $[0, 1]$ according to a density μ_i and a cumulative density M_i . The density μ_i is atomless. We assume that formal and informal productivities (w_i^f and w_i^s) are differentiable with respect to type and denote these derivatives by \dot{w}_i^f and \dot{w}_i^s . It will be useful

⁷Ebert (1992) relies on the optimal control theory to derive the optimal tax when the monotonicity condition is binding. We use the more transparent variational method and develop the optimal bunching condition in the spirit of the Diamond (1998) tax formula.

to denote the growth rates of productivities by $\rho_i^x = \frac{\dot{w}_i^x}{w_i^x}$, $x \in \{f, s\}$. Types are sorted such that the formal productivity is increasing: $\dot{w}_i^f > 0$. We will use the dot notation to write derivatives with respect to type of other variables as well. For instance, \dot{y}_i^f stands for the derivative of formal income with respect to type, evaluated at some type i .

We focus on preferences without wealth effects. Agents' utility function is $U(c, n) = c - v(n)$, where v is increasing, strictly convex and twice differentiable function. We denote the inverse function of the marginal disutility from labor v' by g and the elasticity of labor supply of type i by ζ_i .⁸ Let $V_i(y^f, T)$ be the indirect utility function of an agent of type i whose reported formal income is y^f and who pays a tax T :

$$V_i(y^f, T) \equiv \max_{n^s \geq 0} y^f + w_i^s n^s - T - v\left(\frac{y^f}{w_i^f} + n^s\right). \quad (1.18)$$

In addition to earning the formal income, the agent is optimally choosing the amount of informal labor. Due to concavity of the problem, the choice of n^s is pinned down by the familiar first order condition, modified to allow for the corner solution

$$\min \left\{ v' \left(\frac{y^f}{w_i^f} + n^s \right) - w_i^s, n_i^s \right\} = 0. \quad (1.19)$$

Whenever the formal income y^f is sufficiently high, no shadow labor is supplied. Conversely, sufficiently low formal income leads to informal employment.

The planner chooses a formal income schedule y^f and a tax schedule T in order to maximize a general social welfare function

$$\max_{(y_i^f, T_i)_{i \in [0,1]}} \int_0^1 \lambda_i G\left(V_i(y_i^f, T_i)\right) d\mu_i, \quad (1.20)$$

where G is an increasing and differentiable function and the Pareto weights $\lambda \in [0, 1] \rightarrow \mathbb{R}_+$ integrate to 1.⁹ The budget constraint is the following

$$\int_0^1 T_i d\mu_i \geq E, \quad (1.21)$$

where the net tax revenue needs to cover some fixed expenditures E . Moreover, the tax schedule

⁸Since we abstract from wealth effects, the compensated and uncompensated elasticities coincide. Note that the elasticity is in general an endogenous object, as it depends on labor supply: $\zeta_i = \frac{v'(n_i)}{n_i v''(n_i)}$.

⁹It's easy to relax the assumption of a finite Pareto weight on each type and we are going to do it in the quantitative section, where we consider, among others, the Rawlsian planner.

has to satisfy incentive compatibility

$$\forall_{i,j \in [0,1]} V_i(y_i^f, T_i) \geq V_i(y_j^f, T_j), \quad (1.22)$$

which means that no agent can gain by mimicking any other type. The allocation which solves (1.20) subject to (1.21) and (1.22) is called the second-best or the optimum.

We will describe the optimum by specifying the marginal tax rate of each type. The marginal tax rate is given by the ratio of slopes of the total tax schedule and the formal income schedule

$$t_i = \frac{\dot{T}_i}{\dot{y}_i^f}. \quad (1.23)$$

Intuitively, it describes the fraction of a marginal formal income increase that is claimed by the planner.

1.3.2 Incentive-compatibility

The single crossing property allows the planner in the standard Mirrlees model to focus only on local incentive compatibility constraints. Intuitively, the single-crossing means that, given a constant tax rate, a higher type is willing to earn more than a lower type. The single-crossing in our model means that, holding the tax rate constant, the higher type is willing to earn *formally* more than the lower type.

Assumption 1.1. *A comparative advantage in shadow labor is decreasing with type: $\frac{d}{di} \left(\frac{w_i^s}{w_i^f} \right) < 0$.*

Lemma 1.1. *Under Assumption 1.1, the indirect utility function V has the single crossing property.*

The single-crossing holds when the agents with lower formal productivity have a comparative advantage in working in the informal sector. The single-crossing allows us to replace the general incentive compatibility condition (1.22) with two simpler requirements.

Proposition 1.9. *Under Assumption 1.1, the allocation $(y_i^f, T_i)_{i \in [0,1]}$ is incentive-compatible if and only if the two conditions are satisfied:*

1. y_i^f is non-decreasing in type.
2. If \dot{y}_i^f exists, then the local incentive-compatibility condition holds: $\frac{d}{dj} V_i(y_j^f, T_j) \Big|_{j=i} = 0$.

The utility schedule $V_i(y_i^f, T_i)$ of an incentive compatible allocation is continuous everywhere, differentiable almost everywhere and for any $i < 1$ can be expressed as

$$V_i(y_i^f, T_i) = V_0(y_0^f, T_0) + \int_0^i \dot{V}_j(y_j^f, T_j) dj, \quad (1.24)$$

where

$$\dot{V}_j(y_j^f, T_j) \equiv (\rho_j^f n_j^f + \rho_j^s n_j^s) v'(n_j). \quad (1.25)$$

The single crossing implies that for any tax schedule the level of formal income chosen by a worker is weakly increasing in the worker's type. Hence, assigning a lower income to a higher type would violate incentive compatibility. It is enough to focus just on local deviations: no agent should be able to improve utility by marginally changing the formal earnings. This local incentive-compatibility constraint is equivalent to the familiar condition for the optimal choice of the formal income given the marginal tax rate t_i , allowing for the corner solution

$$\min \left\{ v' \left(\frac{y_i^f}{w_i^f} + n_i^s \right) - (1 - t_i) w_i^f, y_i^f \right\} = 0. \quad (1.26)$$

Note that the formal income may be, and sometimes will be, discontinuous in type. Nevertheless, the indirect utility function preserves some smoothness and can be expressed as an integral of its marginal increments.

Let's call $\dot{V}_i(y_i^f, T_i)$ the *marginal information rent* of type i . It describes how the utility level changes with type. The higher the average rate of productivity growth, weighted by the labor inputs in two sectors, the faster utility increases with type. We will use perturbations in the marginal information rent to derive the optimal tax schedule.

In what follows we will economize on notation of the utility schedule and its slope by suppressing the arguments: $V_i \equiv V_i(y_i^f, T_i)$ and $\dot{V}_i \equiv \dot{V}_i(y_i^f, T_i)$.

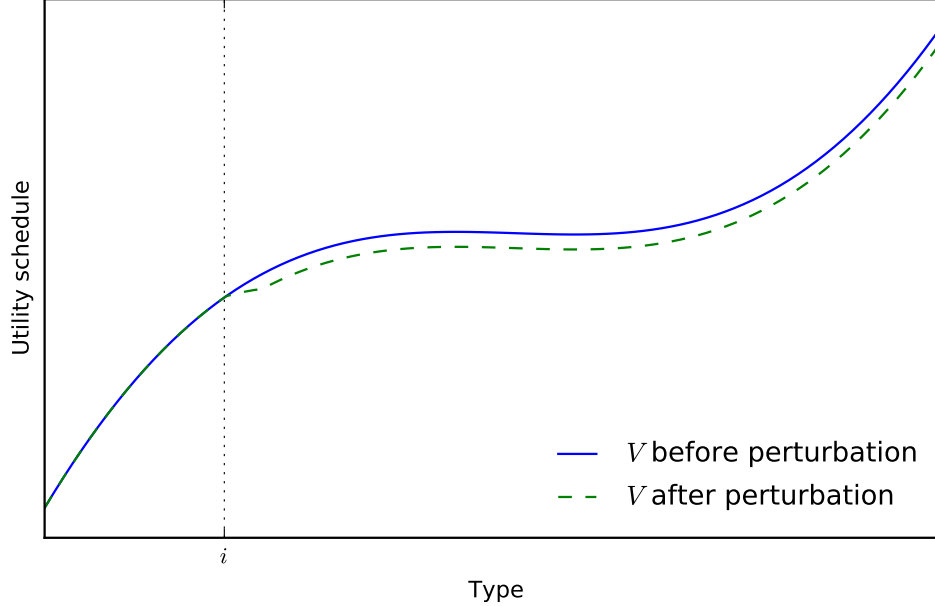
1.3.3 Optimality conditions

First, we solve for the optimum under assumption that the resulting formal income schedule is non-decreasing. Second, we examine when this assumption is justified and show that the existence of the shadow economy make it's violation more likely. Finally, we derive the optimality conditions in the general case.

Interior optimality conditions

We obtain the interior optimality conditions by making sure that the social welfare cannot be improved by perturbing the marginal information rent of any type.¹⁰ A marginal information rent is a slope of the utility schedule at some type i . It can be reduced by increasing tax distortions

¹⁰To the best of our knowledge, [Brendon \(2013\)](#) was the first to use this approach in the Mirrlees model. He also inspired us to express the optimality conditions with endogenous cost terms, although our notation differs from his.

Figure 1.3: Decreasing the marginal information rent of type i 

of this type, which is costly for the budget. On the other hand, by (1.24) such perturbation shifts downwards the entire utility schedule above type i (see Figure 1.3). This shift is a uniform increase of a non-distortionary tax of all types above i . The interior optimality conditions balance the cost of distortions with gains from efficient taxation for each type. Below we present terms that capture the marginal costs and benefits of such perturbations. We derive them in detail in the proof of Theorem 1.1. The shadow economy enters the picture by affecting the cost of increasing tax distortions.

The benefit of shifting the utility schedule of type j without affecting its slope is given by the standard expression

$$N_j \equiv (1 - \omega_j) \mu_j, \text{ where } \omega_j = \frac{\lambda_j}{\eta} G'(V_j). \quad (1.27)$$

A marginal increase of non-distortionary taxation of type j leads to one-to-one increase of tax revenue. On the other hand, it reduces the social welfare, since the utility of type j falls. Following [Piketty and Saez \(2013\)](#) we call this welfare impact the *marginal welfare weight* and denote it by ω_j . Note that welfare impact is normalized by the Lagrange multiplier of the resource constraint η . It allows us to express changes in welfare in the unit of resources. We multiply the whole expression by the density of type j in order to include all agents of this type. We assumed that there are no wealth effects, so the non-distortionary tax does not affect the labor choice of agents. Consequently, the term N_j does not depend on whether type j works informally.

The cost of decreasing some agent's marginal information rent depends on the involvement of this agent in the shadow activity. Types can be grouped into three sets:

$$\begin{aligned} \text{formal workers:} \quad & \mathcal{F} \equiv \left\{ i \in [0, 1] : v' \left(n_i^f \right) > w_i^s \right\}, \\ \text{marginal workers:} \quad & \mathcal{M} \equiv \left\{ i \in [0, 1] : v' \left(n_i^f \right) = w_i^s \right\}, \\ \text{shadow workers:} \quad & \mathcal{S} \equiv \left\{ i \in [0, 1] : v' \left(n_i^f \right) < w_i^s \right\}. \end{aligned}$$

The formal workers supply only formal labor: their marginal disutility from working is strictly greater than their shadow productivity. The marginal workers also supply only formal labor, but their marginal disutility from work is exactly equal to their shadow productivity. A small reduction of formal labor supply of these agents would make them work in the informal sector. Finally, the shadow workers are employed informally, although they can also supply some formal labor.

The formal workers act exactly like agents in the standard Mirrlees model. By increasing distortions, the planner is reducing their total labor supply. The cost of increasing distortions is given by

$$D_i^f \equiv \frac{t_i}{1-t_i} \left(\rho_i^f \left(1 + \frac{1}{\zeta_i} \right) \right)^{-1} \mu_i. \quad (1.28)$$

The cost depends positively on the marginal tax rate. The marginal tax rate tell us how strongly a reduction of the formal income influences the tax revenue. Moreover, the cost increases with the elasticity of labor supply ζ_i and is proportional to the density of the distorted type. D_i^f is endogenous, as it depends on the marginal tax rate.

The perturbation of the marginal information rent works differently for the shadow workers. They supply shadow labor in the quantity that satisfies $v' \left(n_i^f + n_i^s \right) = w_i^s$, which means that their total labor supply n_i is constant. By distorting the formal income, the planner simply shift their labor from the formal to the informal sector. As a result, the cost of increasing distortions does not depend on the elasticity of labor supply, but rather on the sectoral productivity differences,

$$D_i^s \equiv \frac{w_i^f - w_i^s}{w_i^s} \left(\rho_i^f - \rho_i^s \right)^{-1} \mu_i. \quad (1.29)$$

The first term is the relative productivity difference between formal and informal sector. Actually, it's also equal to $\frac{t_i}{1-t_i}$, since the marginal tax rate of these types equalizes the return to labor in both sectors: $(1-t_i) w_i^f = w_i^s$. Hence, as in the case of formal workers, the first term corresponds to the direct tax revenue cost of reduced formal labor supply. The second term describes how effectively the planner can manipulate the agent's marginal information rent by discouraging the formal labor. By the single-crossing assumption, this term is always positive. Again, the density μ_i aggregates the expression to include all agents of type i . Note that D_i^s is exogenous, as it depends

only on the fundamentals of the economy.

The marginal workers are walking a tightrope between their formal and shadow colleagues. If the planner marginally reduces their income, they become the shadow workers. If the planner lifts distortions, they join the formal workers. The cost of changing distortions of these types depends on the direction of perturbation and is equal to either D_i^f or D_i^s .

Having all the cost and benefit terms ready, we can derive the interior optimality conditions. Recall, that by varying the distortions imposed on some type, the planner changes a non-distortionary tax of all types above. In the optimum, the planner cannot increase the social welfare by such perturbations. For the formal workers, this means that

$$\forall_{i \in \mathcal{F}} D_i^f = \int_i^1 N_j dj. \quad (1.30)$$

It is a standard optimality condition from the Mirrlees model, derived first in the quasilinear case by [Diamond \(1998\)](#). The shadow economy does not affect the marginal tax rate of formal agents directly. It may influence them only indirectly, by changing the marginal welfare weights of types above.

For the marginal workers it must be the case that increasing tax distortions is beneficial as long as they work only formally, but it is too costly when they start to supply the shadow labor.

$$\forall_{i \in \mathcal{M}} D_i^s \geq \int_i^1 N_j dj \geq D_i^f \text{ and } y_i^f = w_i^f g(w_i^s). \quad (1.31)$$

The marginal workers do not supply informal labor, but in their case the shadow economy constitutes a binding constraint for the planner. Absent the shadow economy, the marginal tax rates would be set at a higher level. In our model the planner is not willing to do it, because it would push the marginal workers to informal jobs, which is too costly. Formal labor supply of the marginal workers is fixed at the lowest level that leaves them no incentives to work informally.

Recall that the cost of distorting the shadow worker is fixed by the parameters of the economy. Moreover, the benefit of distorting one particular worker, given by (1.27), is fixed as well, since the perturbation of the marginal information rent of i has an infinitesimal effect on the utility of types above. If the planner finds it optimal to decrease the formal income of agent i so much that i starts supplying informal labor, it will be optimal to decrease the formal income all the way to zero, when i works only in the shadow economy:

$$\forall_{i \in \mathcal{S}} \int_i^1 N_j dj > D_i^s \text{ and } y_i^f = 0. \quad (1.32)$$

Note that according to this condition all shadow workers are bunched together at zero formal

income.¹¹

The optimality conditions (1.30)-(1.32) determine the slope of the utility schedule at each type. What is left is finding the optimal level. Suppose that the planner varies the tax paid by the lowest type, while keeping all the marginal rates fixed. Optimum requires that such perturbation cannot improve welfare:

$$\int_0^1 N_j dj = 0. \quad (1.33)$$

Definition. Conditions (1.30)-(1.33) are called the *interior optimality conditions*. The allocation (y^f, T) consistent with the interior optimality conditions is called the *interior allocation*. Specifically, y^f is called the *interior formal income schedule*.

The interior conditions are necessary for the optimum as long as they don't imply a formal income schedule which is locally decreasing. They become sufficient, if they pin down a unique allocation. This happens when the cost of distortions is increasing in the amount of distortions imposed. When that is the case, the planner's problem with respect to each type becomes concave. Theorem 1.1 provides regularity conditions which guarantee it.

Assumption 1.2. (i) The elasticity of labor supply $\frac{v'(n)}{nv''(n)}$ is non-increasing in n . (ii) The ratio of sectoral growth rates is bounded below $\forall_i \frac{\rho_i^s}{\rho_i^f} > -\zeta_i^{-1}$.

Theorem 1.1. Under Assumption 1.1, if all interior formal income schedules are non-decreasing, the interior optimality conditions are necessary for the optimum. Under Assumptions 1.1 and 1.2, there is a unique interior formal income schedule. If it is non-decreasing, the interior optimality conditions are both necessary and sufficient for the optimum.

When do the interior conditions fail?

The interior allocation is incentive-compatible and optimal if it leads to formal income that is non-decreasing in type. In the standard Mirrlees model formal income is decreasing if the marginal tax rate increases too quickly with type. However, in virtually all applications of the standard Mirrlees model this is not a problem, as the conditions under which the interior tax rate increases that fast are rather unusual.¹² The shadow economy gives rise to another reason for non-monotone interior

¹¹Notice that we could replace the strict inequality with a weak one in (1.32), and conversely regarding the left inequality in (1.31). In words, when the cost of distorting some marginal worker is exactly equal to the benefit, then this worker could equally well be a shadow worker, with no change in the social welfare. It means that whenever the curves D_i^s and $\int_i^1 N_j dj$ cross, the optimum is not unique, since we could vary allocation of the type at the intersection. Since such a crossing is unlikely to happen more than a few times, we do not consider this as an important issue. We sidestep it by assuming that the planner introduces distortions only when there are strictly positive gains from doing so. Consequently, our notion of uniqueness of optimum should be understood with this reservation.

¹²Probably simplest way to construct an example of locally decreasing formal income schedule is to assume a bimodal productivity distribution, with very low density between the modes.

formal income. In the interior allocation all shadow workers have zero formal income. Hence, if there is any worker with positive formal income with a type lower than some shadow worker, the formal income schedule will be locally decreasing. It turns out that this second reason makes the failure of the interior allocation much more likely. In Proposition 1.10 below we provide the sufficient conditions for the formal income to be non-decreasing. Then we discuss the two cases in which the shadow economy leads to the failure of the interior optimality conditions.

Assumption 1.3. (i) The social welfare function is such that $G(V) = V$, λ_i is non-decreasing in type for $i > 0$. (ii) The ratio $\frac{1}{\rho_i^f} \frac{\mu_i}{1-M_i}$ is non-decreasing in type. (iii) The elasticity of labor supply is constant: $\forall_i \zeta_i = \zeta$. (iv) The ratio of sectoral growth rates $\frac{\rho_i^s}{\rho_i^f}$ is non-decreasing in type.

Proposition 1.10. Under Assumptions 1.1, 1.2 and 1.3, the unique interior formal income schedule is non-decreasing.

First, notice that we make sure that the interior formal income schedule is unique (Assumption 1.2). Simultaneously, it implies that the formal income of the marginal workers is non-decreasing. Assumptions 1.3(i) - 1.3(iii) make sure that the marginal tax rate of formal workers is non-increasing in type, which in turn implies that the formal income of these workers is non-decreasing. These conditions are familiar from the standard Mirrlees model. Assumption 1.3(i) is satisfied by the utilitarian or Rawlsian social welfare function, while Assumption 1.3(ii) is a weaker counterpart of the usual monotone hazard ratio requirement.¹³

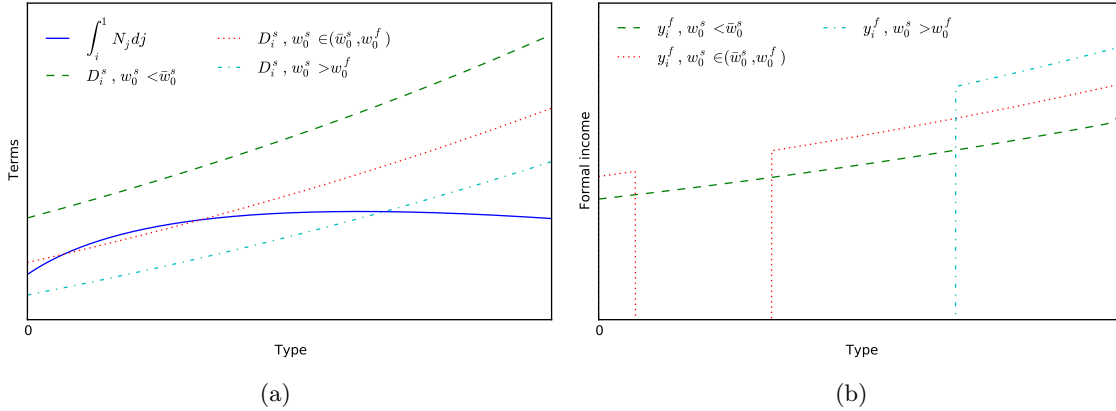
Finally, we have to make sure that all shadow workers, if there are any, are at the bottom of the type space. By (1.32) it means that the marginal cost of distorting the shadow worker D_i^s can cross the marginal benefit $\int_i^1 N_j dj$ at most once and from below. It is guaranteed jointly by conditions 1.3(i), 1.3(ii) and the new requirement 1.3(iv) which says that the ratio of sectoral productivity growth rates is non-decreasing. In addition to assuring the optimality of the interior allocation, Assumption 1.3 imply also that sets \mathcal{S}, \mathcal{M} and \mathcal{F} , if non-empty, can be ordered: the bottom types are the shadow workers, above them are the marginal workers, and the top types are formal.

Assumption 1.2 makes sure that the D_i^s curve crosses the $\int_i^1 N_j dj$ curve at most once. Let's see how the relaxation of some of its elements make these curves cross more than once. In Example 1 we relax the assumption on the social welfare function and in Example 2 we allow the non-monotone ratio of sectoral growth rates.

Example 1. (i) The social welfare function is such that $G(V) = V$, the Pareto weights λ_i are continuous in type and satisfy $\lambda_0 > 2$. (ii) The distribution of types is uniform. (iii) The elasticity

¹³We can express the distribution of types as a function of formal productivity rather than type. Then the density is $\bar{\mu}(w_i^f) = \frac{\mu_i}{w_i^f}$ and cumulative density is $\bar{M}(w_i^f) = M_i$. Hence, assumption 1.3(ii) means that $\frac{w_i^f \bar{\mu}(w_i^f)}{1-\bar{M}(w_i^f)}$ is non-decreasing. For instance, any Pareto distribution of formal productivity satisfies this assumption.

Figure 1.4: A failure of the interior allocation due to increasing benefit of distortions $\int_i^1 N_j dj$ (Example 1).



of labor supply is constant: $\forall_i \zeta_i = \zeta$ and $v'(0) = 0$. (iv) The ratio of sectoral growth rates $\frac{\rho_i^s}{\rho_i^f}$ is fixed. (v) Assumptions 1.1 and 1.2 are satisfied.

Lemma 1.2. *In Example 1 there is a threshold $\bar{w}_0^s \in (0, w_0^f)$ such that if $w_0^f > w_0^s > \bar{w}_0^s$ the interior formal income schedule **is not** non-decreasing.*

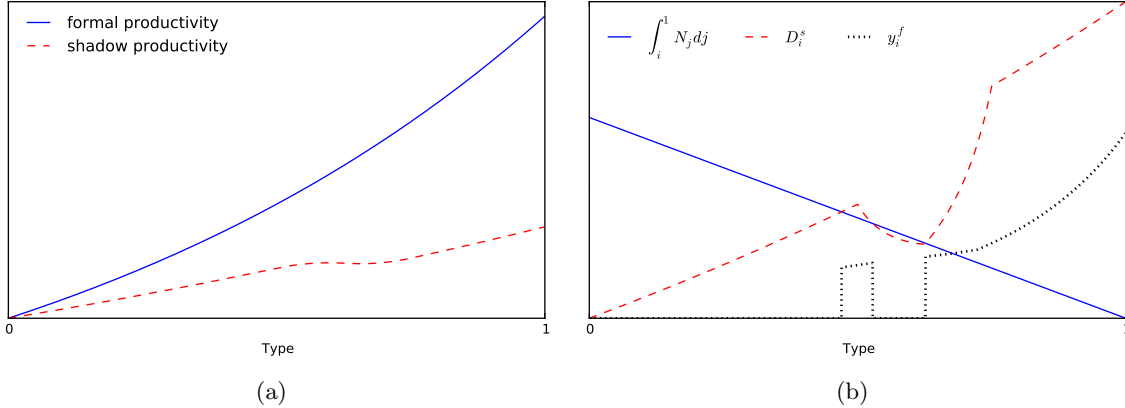
Example 1 violates Assumption 1.3 (i), which allows the $\int_i^1 N_j dj$ term to be initially increasing in type.¹⁴ Both terms D_i^s and $\int_i^1 N_j dj$ are increasing at 0, but $\int_i^1 N_j dj$ term increases faster. If $w_0^f > w_0^s$, then the distortion cost at type 0 is greater than the benefit and the bottom type works formally. If the gap between w_0^f and w_0^s is sufficiently small (smaller than $w_0^f - \bar{w}_0^s > 0$), D_i^s curve will cross the benefit curve at some positive type (see Figure 1.4). Consequently, the agents above the intersection will work in the shadow economy. Since these agents have no formal income, the formal income schedule is locally decreasing.

Example 2. (i) The social welfare function is Rawlsian: $\forall_{i>0} \lambda_i = 0$. (ii) The distribution of types is uniform. (iii) The elasticity of labor supply is constant: $\forall_i \zeta_i = \zeta$. (iv) The growth rate of formal productivity is fixed, while the growth rate of shadow productivity is decreasing for some types. (v) Assumptions 1.1 and 1.2 are satisfied.

Example 2 satisfies all the requirements of Proposition 1.10 apart from the non-decreasing sectoral growth rates ratio assumption. In panel (a) of Figure 1.5 we can see that the growth rate of shadow productivity decreases around the middle type and then bounces back. It is reflected in

¹⁴The Pareto weights integrate to 1 over the type space, so they have to be lower than or equal to 1 for some types above 0. Since these weights are continuous and $\lambda_0 > 2$, they will be decreasing for some type above 0, violating 1.3(i).

Figure 1.5: A failure of the interior allocation due to non-monotone ratio of productivity growth rates (Example 2).



the marginal cost of distorting shadow workers D_i^s (panel (b)). We chose the parameters such that the fall is substantial, making the D_i^s curve cross the $\int_i^1 N_j dj$ curve three times. Consequently, the formal income first increases, then decreases to 0 once the D_i^s crosses $\int_i^1 N_j dj$ for the second time. This example shows that even minor irregularities in the distribution of productivities can make the interior allocation not implementable.

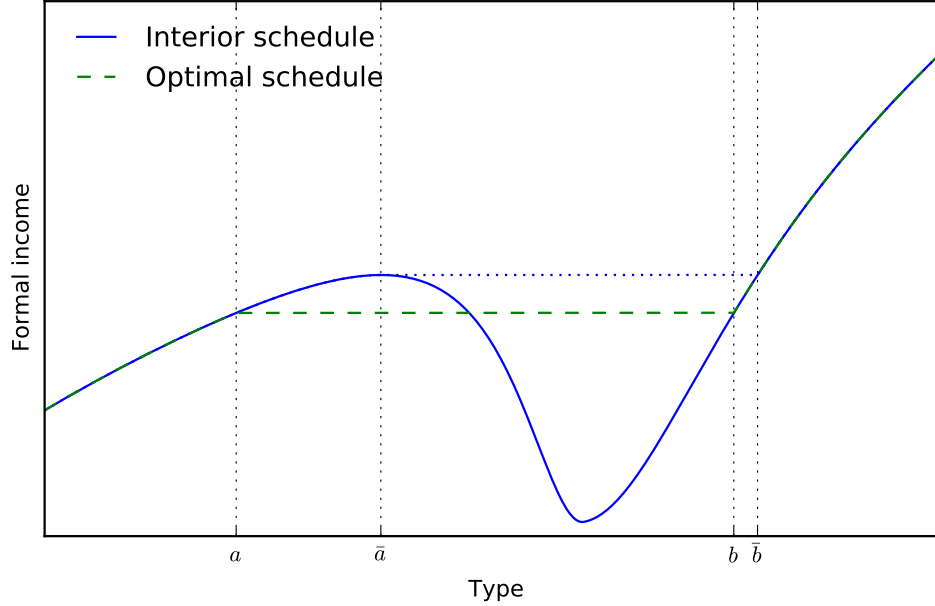
Optimal bunching

Whenever the interior formal income schedule is decreasing for some types, the interior allocation is not incentive-compatible and hence is not optimal. Ebert (1992) and Boadway, Cuff, and Marchand (2000) applied the optimal control theory to overcome this problem. In contrast to these papers, we derive the optimal bunching condition with the intuitive variational argument and express it in the spirit of the Diamond (1998) optimal tax formula. What we are going to do is essentially “ironing” the formal income schedule whenever it is locally decreasing (see Figure 1.6). The ironing was originally introduced by Mussa and Rosen (1978) in a solution to the monopolistic pricing problem when the monotonicity condition is binding.

Suppose that the interior formal income schedule \bar{y}^f is decreasing on some set of types, beginning with \bar{a} . Decreasing formal income is incompatible with the incentive-compatibility. We can regain incentive-compatibility by lifting the schedule such that it becomes overall non-decreasing and flat in the interval $[\bar{a}, \bar{b}]$ (see Figure 1.6). Since types $[\bar{a}, \bar{b}]$ have the same formal income, they are bunched and cannot be differentiated by the planner. Such bunching is implemented by a discontinuous jump of the marginal tax rate.

The flattened schedule is incentive-compatible. However, generally it is not optimal. By marginally

Figure 1.6: Ironing the formal income schedule



decreasing formal income of type \bar{a} the planner relaxes the binding monotonicity constraint and can marginally decrease the formal income of all types in the interval (\bar{a}, \bar{b}) . This perturbation closes the gap between the actual formal income and its interior value for the positive measure of types. On the other hand, the cost of perturbation is infinitesimal: it is a distortion of one type \bar{a} . This perturbation is clearly welfare-improving, starting from the flattened interior schedule. Below we find the optimal bunching condition by making sure that the perturbation is not beneficial at the optimal income schedule.

Suppose that an interval of agents $[a, b]$ is bunched. Let's marginally decrease the formal income of agents $[a, b)$ and adjust their total tax paid such that the utility of type a is unchanged. In this way we preserve the continuity of the utility schedule. However, since the other bunched agents have a different marginal rate of substitution between consumption and income, this perturbation will decrease their utility. We normalize the perturbation such that we obtain a unit change of the utility of the highest type in the bunch. The total cost of this perturbation is given by

$$D_{a,b} \equiv (t_a + \mathbb{E} \{ \Delta MRS_i \omega_i | b > i \geq a \}) \frac{M_b - M_a}{t_{b^+} - t_{a^-}}, \quad (1.34)$$

$$\text{where } \Delta MRS_i = \frac{v'(n_a)}{w_a^f} - \frac{v'(n_i)}{w_i^f}.$$

The expression within the brackets is an average impact of a unit perturbation of the formal income.

The brackets contain two components: a fiscal and a welfare loss. The fiscal loss from reducing the formal income of each bunched agent is the marginal tax rate below the kink. The welfare loss is an average marginal welfare weight in the bunch corrected by a discrepancy of the marginal rate of substitution of a given type from type a . The larger ΔMRS_j is, the more type j suffers from the perturbation. Note that ΔMRS_b is just equal $t_{b+} - t_{a-}$.¹⁵ Hence, in order to normalize the perturbation to have a unit impact on utility of type b , we divide the brackets by $t_{b+} - t_{a-}$. We aggregate this average effect by multiplying it by the mass of bunched types.

The benefit of this perturbation comes from the reduced utility of types above b and is the same as in the interior case. The optimality requires that

$$\min \left\{ \int_b^1 N_j dj - D_{a,b}, y_a^f \right\} = 0. \quad (1.35)$$

Note that the optimality condition involves a corner solution when $y_a^f = 0$. It corresponds to the situation in which the bunched workers don't work formally at all.

The optimality condition (1.35) is influenced by the shadow economy again through the cost of distortion. If some worker i in the bunch $[a, b)$ supplies shadow labor, then the difference in the marginal rate of substitution for this worker is given by $\Delta MRS_i = \frac{v'(n_a)}{w_a^f} - \frac{w_i^s}{w_i^f}$.

Theorem 1.2 combines all the optimality conditions.

Theorem 1.2. *Under Assumption 1.1, the optimal allocation satisfies (1.33) and at each level of formal income one of the three mutually exclusive alternatives hold:*

- *there is no type that reports such formal income,*
- *there is a unique type whose allocation satisfies the interior optimality conditions (1.30)-(1.32),*
- *there is a bunch of types whose allocation satisfy the optimal bunching condition (1.35).*

Although we managed to characterize the full set of optimality conditions, the interior conditions are generally easier to use. Below we show that the interior allocation, even if not incentive-compatible, are a good predictor of which agents optimally work in the shadow economy.

Assumption 1.4. *(i) G is a concave function. (ii) ρ_i^f , ρ_i^s , μ_i and λ_i are continuous in type.*

Proposition 1.11. *Under Assumptions 1.1, 1.2 and 1.4, all the types that supply shadow labor in the interior allocation remain the shadow workers in the optimum.*

¹⁵The marginal tax rate discontinuously increases at the kink. By t_{a-} we denote the tax rate below the kink and by t_{b+} the tax rate above the kink.

1.3.4 Summary of results

Which agents should work in the shadow economy?

Corollary 2. *Suppose that $v'(0) = 0$. Under Assumptions 1.1, 1.2 and 1.4 type i optimally works in the shadow economy if*

$$\mathbb{E} \{1 - \omega_j | j > i\} \geq \frac{w_i^f - w_i^s}{w_i^f} \left(-\frac{d}{di} \left(\frac{w_i^s}{w_i^f} \right) \right)^{-1} \frac{\mu_i}{1 - M_i}. \quad (1.36)$$

This condition is both necessary and sufficient if the interior allocation is incentive-compatible.

The inequality (1.36) compares the gains from efficient taxation of all types above i with the cost of distorting type i , when this type is at the edge of joining the shadow economy. A type i is likely to optimally work in the shadow economy if the planner on average puts a low marginal welfare weights on the types above i , the relative productivity loss from moving to informal employment is low and the density of distorted types is low in comparison to the fraction of types above. Finally, the shadow employment is more likely if the comparative advantage of working in the shadow sector $\frac{w_i^s}{w_i^f}$ is quickly decreasing with type. It means that higher types have less incentives to follow type i into the shadow economy. We assume $v'(0) = 0$ so that we do not have to worry about some types not supplying any labor at all.

Note that with the Rawlsian planner the inequality (1.36) is just a continuous equivalent of the condition (1.10) from the simple model.

The optimal tax rates. Let's focus on agents that supply some formal labor and are not bunched at the kinks of the tax schedule. These types never supply informal labor. The optimal tax formula is

$$\frac{t_i}{1 - t_i} = \min \left\{ \frac{w_i^f - w_i^s}{w_i^s}, \rho_i^f \left(1 + \frac{1}{\zeta_i} \right) \frac{1 - M_i}{\mu_i} \mathbb{E} (1 - \omega_j | j > i) \right\}. \quad (1.37)$$

The shadow economy imposes an upper bound on the marginal tax rate. The bound (the left term in the min operator of (1.37)) is such that the tax rate equalizes the return from formal and informal labor - it is the highest tax rate consistent with agents working in the formal sector.

If the bound is not constraining the planner, then the tax rate should be set according to [Diamond \(1998\)](#) formula (the right term in the min operator of (1.37)). The expectations describe the average social preferences towards all types above i . In general, the less the planner cares about increasing utility of the types above i , the higher t_i will be. If the Pareto weights increase with type or G is a strictly convex function, this term may become negative, leading to negative marginal tax rates,

as explained by [Choné and Laroque \(2010\)](#). Since the sign of the tax rate is ambiguous, below we describe how the other terms influence its absolute value. The optimal tax rate increases in absolute value when the growth rate of formal productivity with respect to type is high. If the planner is redistributive and types above i are much more productive than types below, it is optimal to set a high tax rate. The tax rate decreases with elasticity of labor supply ζ_i , as it makes workers more responsive to the tax changes. The ratio $\frac{1-M_i}{\mu_i}$ tells us how many agents will be taxed in a non-distortionary manner relative to the density of distorted agents. If this ratio is high, the gain from increasing tax rates relative to the cost will be high as well.

Optimal bunching. Bunching may arise at the bottom of the formal income distribution, resulting in de facto exclusion from the formal labor market. Bunching may also appear at a positive level of formal income, which implies a kink in a tax schedule. All workers who supply shadow labor are subject to bunching, though not necessarily at the same tax kink. Some workers supplying only formal labor can be found at the kinks as well. The formal income schedule at which the kink is located is determined by

$$\frac{t_{a-}}{t_{b+} - t_{a-}} = \frac{1 - M_b}{M_b - M_a} \mathbb{E} \{ 1 - \omega_j | j \geq b \} - \mathbb{E} \left\{ \frac{\Delta MRS_i}{\Delta MRS_b} w_i \middle| b > i \geq a \right\}, \quad (1.38)$$

where a and b are respectively the lowest and the highest type bunched at the kink. Note that both t_{a-} and t_{b+} , the tax rates below and above the kink, are set according to (1.37). The location of the kink is determined by the trade-off between tax and welfare losses from the bunched agents and the tax revenue gains from the efficient taxation of agents above the kink.

1.4 Measuring shadow and formal productivities

To assess the practical relevance of our theoretical results we proceed to look at the empirical counterparts of the building blocks of our theory. We focus on a developing economy with a large shadow sector: Colombia.¹⁶ In this section we empirically estimate the three key objects of the model: the formal productivity (w_i^f), the informal productivity (w_i^s) and the distribution of types (μ_i). In section 1.5 we use our estimates to analyze how the existence of the shadow economy shapes the optimal tax scheme in Colombia.

Colombia is a case that suits itself very well to take our theory to the data, because the shadow economy is large and we can actually observe the total income of individuals, both if formal or shadow, through survey data. Household surveys reveal information about shadow income without

¹⁶58% of the workers are part of the shadow economy according to our estimates.

making it usable by the authorities to levy taxes.¹⁷ Furthermore, Colombian regulation makes it easy to infer shadow and formal income from questions about total income, and from the type of affiliation of the worker to the social security system.

In the model, w_i^f and w_i^s correspond to the pre-tax (real) income for one unit of labour for individual of type i in each sector, and μ_i is the density of such type. Therefore, we have one-dimensional heterogeneity across individuals. Our empirical strategy is to replicate such one-dimensional heterogeneity by using a factor that comprises information of the worker and job characteristics, such as the education level and the task done on the job. The identification assumptions is that the pre-tax hourly wage recorded on the surveys is a noisy signal of the productivities in each sector and that the productivities themselves are a linear function of the factor we employ.

The weights that are used to construct the factor and the parameters that map productivities to wages are jointly estimated to maximize the explanatory content of the factor over wages. Indeed, the factor we obtain can explain most of the variability of wages in both sectors. Nevertheless, the factor cannot account for the income dispersion of the top earners and the gap with respect to the rest of the population. We extend our identification strategy by estimating a Pareto distribution for the wages of top earners in the formal sector.

We find that both productivity estimates are increasing in type (the factor) and that the single-crossing property is satisfied. Specifically, the wedge between the productivity levels of each sector is almost zero for the least productive agents and increases rapidly as the formal productivity increases. The main novelty of this section is that we assess the differences between the formal and the shadow economy at the worker level, controlling for the sorting of workers. Productivity as measured in [La Porta and Shleifer \(2008\)](#) can come also from the worker characteristics and not only from the type of firms or jobs in each sector. With our approach we are able to discuss the wage differential across sectors for a given worker and job. On the other hand, the mapping of our estimates to productivity levels depends on the structure of the labor and goods market, because we rely on data on wages rather than quantities produced or profits of the firm; as those other studies do. For the purposes of this paper this is not important since our object of interest is the income of the worker in each sector. Our results can shed light on the productive structure of the two sectors once the link between wages and productivity is specified.¹⁸

The remaining of this section is organized as follows: first, we present the data and show how we identify informal workers. Second, the empirical specification is presented and last, the results are shown and discussed.

¹⁷Households are explicitly guaranteed that their answers have no legal implications and cannot be used against them by any government agency.

¹⁸For example, if it is assumed that there is perfect competition on the labor market, then our measure corresponds directly to the worker's marginal productivity. With the additional assumption of a production function with constant returns to scale, our measure also reflects the average productivity of the worker.

1.4.1 Data

Our source of information is the household survey (ECH by the Spanish acronym) collected on a monthly basis by the official statistical agency in Colombia (DANE). Our sample is for the year 2013 and comprises 170.000 observations of workers. The sample includes personal information such as age, gender, years of education and also labor market related variables including hours worked, number of jobs, type of job, income sources and social security affiliation. All of the information is self-reported by the worker.

The variables we use from the survey can be grouped into 4 categories: worker characteristics, job characteristics, worker-firm relationship and social security status. A linear combination of the variables in the first three categories is used to construct a factor that captures the variability of wages. The fourth category is used to classify individuals as formal or informal workers. Below we provide a brief description of the variables included in each category, for more detailed information see Appendix 1.6.

Worker characteristics capture the type of worker. They include: age, gender, education level and work experience in previous jobs.

Job characteristics describe the type of job and task that the worker does. The variables included are: number of workers in the firm (size), industry to which the firm belongs, geographical location of the firm and the task the worker has to do.

Worker-firm relationship involves the information about the type of contract and the wage determination. The variables included here are: The wage of the worker, number of working hours, the length of the match, whether the worker is hired through an intermediary firm and whether the worker belongs to a union.

Social security status determines whether the worker is affiliated to social security in its different dimensions, and the type of affiliation. The variables included are: affiliation to the health system, the pension system and the labor accidents insurance, as well as who pays for the affiliation to each component.

Classification of workers into formal and shadow workers

Colombian regulation provides for labour tax payments (payroll taxes) and the affiliation to social security to be done jointly. Therefore, the affiliation status to the social security system reveals whether the worker's income is taxed and observed by the government, or shadow. We identify a formal worker as a worker affiliated through his own job to all three main components of labor protection: the health security system, the pension system and the accidents insurance policy. With this criteria we estimate that around 58% of the Colombian workers operate in the shadow sector.

When identifying the sector to which the worker belongs we can incur in type I and type II errors, which are respectively: to classify a worker as shadow when he is formal; and to classify a worker as formal when he is shadow. The type I error is not relevant as the affiliation to the social security system is itself a tax on workers, so any worker not affiliated to the system is by definition avoiding labor taxes. On the other hand, there could be shadow workers that decide to register to social security and pay the corresponding contributions, since the affiliation through the alternative subsidized system is mean-tested¹⁹ and they might be not eligible. The incentive for a shadow worker to register and pay is therefore being covered by the health insurance. On the other hand, what induces these workers to remain shadow and misreport their income is paying a lower social contribution and a consequently lower payroll and income tax. We find that by applying the more stringent criterion that requires affiliation not only to the health but also to the pension system and the accidents insurance policy we are able to mitigate the possibility of identifying a shadow worker that registers to social security as a formal worker, as observations with large deviations between the statutory contributions and the actual contributions tend to be for workers that were only affiliated to one or two of the social security provisions (primarily health) but typically not to the accidents insurance.

Finally we could also face the case of a formal worker paying all contributions to the social security system (and being thus classified as formal) but hiding from the government part of his income. This type of worker does pay taxes, but pays less than the amount imposed by the statutory tax imposes. In the case of employees this possibility is mitigated, due to the fact that the firm or the employer are third parties reporting the worker's income and paying the corresponding taxes to the government.²⁰ The self-employed workers active in the formal sector are also constrained in their income misreporting, since their contractors are the third party in charge to pay the *honorary* tax to tax authorities belong to the formal sector. In conclusion, we believe that these features of the Colombian employment reality allow us to follow the structure of the model by defining tax evasion as working in the shadow economy, while setting aside the aspect of hiding fractions of formal labor income.

Colombian labor tax scheme

The main components of the Colombian tax/transfers scheme associated with formal labor income are income taxes, social insurance (payroll) taxes and transfers. First we describe the individual income tax, then the payroll taxes and then the transfers and subsidies. Using this tax scheme we proceed to compute the pre-tax income from the reported income by households and consequently the effective tax rates.

¹⁹The housing quality of the recipient is also considered as a criterion to be enrolled of the subsidized system

²⁰See for example Kleven, Kreiner, and Saez (2015) for an exploration of the agency role of firms for the implementation of labor taxes and a discussion of the greater tax enforcement when there is third party reporting.

OPTIMAL REDISTRIBUTION WITH A SHADOW ECONOMY

The individual income tax is a progressive tax payable once per year over the total income of one calendar year. The tax is determined by income brackets, and within each of them a fixed amount is paid. The first bracket on which the tax is different from zero starts at 22,219 dollars (annual income in 2013 dollars). The tax rate is increasing across brackets and at the last bracket it reaches 27%.

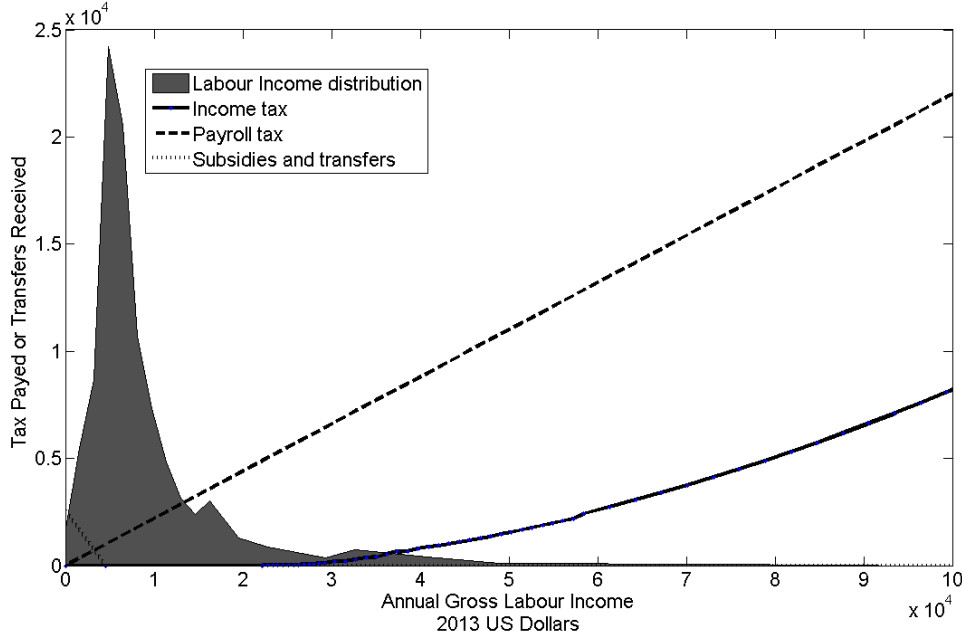
The social insurance taxes are the payroll tax and the health system contribution. For the case of employees these taxes are paid jointly with the employer; each of the two parties paying a specified fraction. The sum of both (irrespective of who is in charge of making the payment) corresponds to a flat tax rate of 22%.

Finally, the bulk of welfare transfers and subsidies in Colombia are granted according to a centralized system that assigns to each household registered in the system a certain score on an index which evaluates needs, life standards, and economic status. The index ranges from 0 to 100, and a series of different welfare programs use it to assign subsidies and transfers, each one according to its own threshold. Part of the questionnaire used to compile the index refers to income of the household. Households have the incentives to misreport income, shadow workers can potentially misreport income while formal workers can be spotted by the system as the reports are crosschecked with the government tax agency. We take an average household that belongs to the subsidized system (meaning the index score is low) (SISBEN) and compute the total transfers it is entitled on that year by the main social programs available. We calculate that those transfers for a household with no formal income could be as large as 2000 dollars per year and reduce to zero for an average household with a full time formal job.

Figure 1.7 presents the tax scheme decomposed in the three elements discussed and the pre-tax income distribution recovered from reported income and the tax scheme. We see that transfers are an important source of income for the poorer households and that the income tax affects a small fraction of total households.

We have focused on the taxes directly associated with labor income. We do not consider, as they are not part of the instruments we consider in the model, the excise taxes and the corporate income taxes (or taxes over capital gains). If we take that excise taxes are only charged over goods produced in the formal sector and that firms in the formal and shadow economy compete for the same markets then we have that the tax will completely fall on the worker of the formal economy. We leave for further research the possibility of using excise taxes in a setup where the link between goods taxation and labor income has more structure to be analyzed. With our approach we focus exclusively on the taxes and transfers that have a direct link with labor income.

Figure 1.7: The Colombian Labor Tax Scheme



Measuring Income and Wages

Our analysis assumes that all payroll taxes and social security contributions irrespectively of who is administratively charged for the tax are a burden on the worker income. A labor tax that has to be paid by the employer is assumed to be translated in a lower wage for the worker.²¹ The workers report their monthly income and the hours worked. To this reported income we input payments that formal workers are entitled to but which are done in a different frequency and are not recorded for the month the survey was conducted. Furthermore, note that we do not include the pension and unemployment insurance contributions as part of the tax burden but we do include them as part of the total income of the worker.

The hourly wage is computed then as the total income divided by the numbers of hours worked. If the worker is a shadow worker we denote it by \tilde{w}_i^s and if it is formal then is denoted by \tilde{w}_i^f . These is the key variable that we are going to map to the productivity levels w_i^s and w_i^f described in the model.

²¹This is a standard assumption for pretax income computations. The Congressional Budget Office in the US uses the same assumption to compute the effective tax rates.

1.4.2 Empirical specification

The logarithm of both productivities (w_i^f and w_i^s) can be written as a function of a single factor F_i as follows

$$\log(w_i^f) = \gamma_0^f + \gamma_1^f F_i \quad (1.39)$$

$$\log(w_i^s) = \gamma_0^s + \gamma_1^s F_i \quad (1.40)$$

where γ_0^j, γ_1^j characterize the linear function in sector $j \in \{f, s\}$. We set $\gamma_1^f = 1$ without loss of generality, given that this will just rescale the factor. The factor is a linear combination of a set of n variables contained in vector X_i with weights given by the vector β . Then we have that

$$F_i = \beta X_i \quad (1.41)$$

The proxy we have for the model productivities are the wages of workers \tilde{w}_i^j in each sector j , then we have that²²

$$\log(\tilde{w}_i^f) = \log(w_i^f) + u_i^f \quad (1.42)$$

$$\log(\tilde{w}_i^s) = \log(w_i^s) + u_i^s \quad (1.43)$$

where u_i^f and u_i^s are random variables with mean zero. Wages are drawn from a probability distribution where the key location parameters are w_i^f and w_i^s , the theoretical concepts in our analysis. In the theoretical analysis we abstract from the underlying variance of the distribution and focus on the limit when it tends to zero. The model is a static economy so we are not concerned with short term variations of wages but rather on the distribution of the location parameters across the population.

Combining equations (1.39) to (1.43) we get the specification of the empirical model that corresponds to

$$\log(\tilde{w}_i) = \gamma_0^f + I_i(\gamma_0^s - \gamma_0^f) + (1 + I_i(\gamma_1^s - 1))\beta X_i + u_i \quad (1.44)$$

where I_i is an indicator function that takes the value of 1 if type i works in the shadow economy and $u_i = I_i u_i^s + u_i^f$. We estimate (1.44) by non-linear least squares.

²²Note that, as discussed earlier, w_i^j is only observed if type i works in sector j .

Ordering of agents and estimated productivities

Note the estimate of parameter a as \hat{a} . We proceed to order the individuals in our sample with indexes $i \in [0, 1]$ such that $i < i' \iff \hat{\beta}X_i < \hat{\beta}X_{i'}$. We compute the index of each individual using the following formula

$$i = \frac{\hat{\beta}X_i - \min_{i'}\{\hat{\beta}X_{i'}\}}{\max_{i'}\{\hat{\beta}X_{i'}\}}$$

that is just rescaling the factor using the minimum and the maximum values it takes in the sample. The estimated productivities of each type i then correspond to

$$\hat{w}_i^f = \exp\left\{\hat{\gamma}_0^f + \hat{\beta}X_i\right\} \quad (1.45)$$

$$\hat{w}_i^s = \exp\left\{\hat{\gamma}_0^s + \hat{\gamma}_1^s \hat{\beta}X_i\right\}. \quad (1.46)$$

Single-crossing condition

The single-crossing condition states that the ratio w_i^f/w_i^s is increasing in type. Using (1.45) and (1.46) this ratio can be written as

$$\frac{\hat{w}_i^f}{\hat{w}_i^s} = \exp\left\{\hat{\gamma}_0^f - \hat{\gamma}_0^s\right\} \exp\left\{(1 - \hat{\gamma}_1^s) \hat{\beta}X_i\right\}$$

Then, if $\hat{\gamma}_1^s < 1$ holds, the single-crossing condition is satisfied. Recall that we standardized to 1 the marginal (percentile) increase of formal productivity to a marginal increase in the factor. Therefore, this condition states that a marginal increase in the factor has to imply a lower marginal increase in shadow than in formal productivity.

Top income earners

We standardized the time available for labor in a year equal to 1 and therefore we can interpret \tilde{w}_i^j as the income of worker i for full time work at sector j , then \hat{w}_i^f corresponds (on average) to the maximum income that type i can achieve. Nevertheless, some income observations are above the maximum value implied by the factor for the most productive worker working full time. That is, there could be labor income observations y_i that satisfy

$$y_i > \max_{i'}\{\hat{w}_{i'}^f\} = \hat{w}_1^f \quad (1.47)$$

We classify the individuals that satisfy this criterion as top earners. These are individuals with a very large wage premium that cannot be accounted for with our benchmark specification and for

which the wage does not seem to have the same relationship with the factor as for the rest of the population.

To characterize with more accuracy this behavior at the top of the income distribution we estimate the upper tail of the productivity distribution by fitting a Type I Pareto distribution for the gross wage \tilde{w} of top earners. The support of the distribution is given by $[\hat{w}_1^f, \infty)$ and the shape parameter is estimated by maximum likelihood.

A final adjustment has to be made to the index of agents. To fit the top earners in the type space $[0, 1]$ we compress the indexes on non-top earners to the interval $[0, k]$ and top earners are assigned to $[k, 1]$ and ordered by their gross wage.

Distribution of types

The assignment of indexes for each observation and their corresponding sampling weights implies a discrete distribution of workers (non-top earners). The continuous distribution of types is obtained by a kernel density estimation with a linear interpolation at the evaluation points. The estimated kernel distribution gives us the distribution of types in the interval $[0, k]$.

For top earners we have a Pareto distribution for productivities with the support $[\max_i \{\hat{w}_i^f\}, \infty)$ but this distribution can be replicated by different types distributions in $[k, 1]$ at the types space, provided that the formal productivities w_i^f for $i \in [k, 1]$ are adjusted accordingly. This phenomenon does not occur with non-top earners because their productivity profiles are given by our parametric model.

There are two requirements that the distribution of types and productivity profiles of top earners satisfy always: the total mass of the distribution has to coincide with the mass of top earners and that $\lim_{i \rightarrow 1} w_i^f = \infty$.

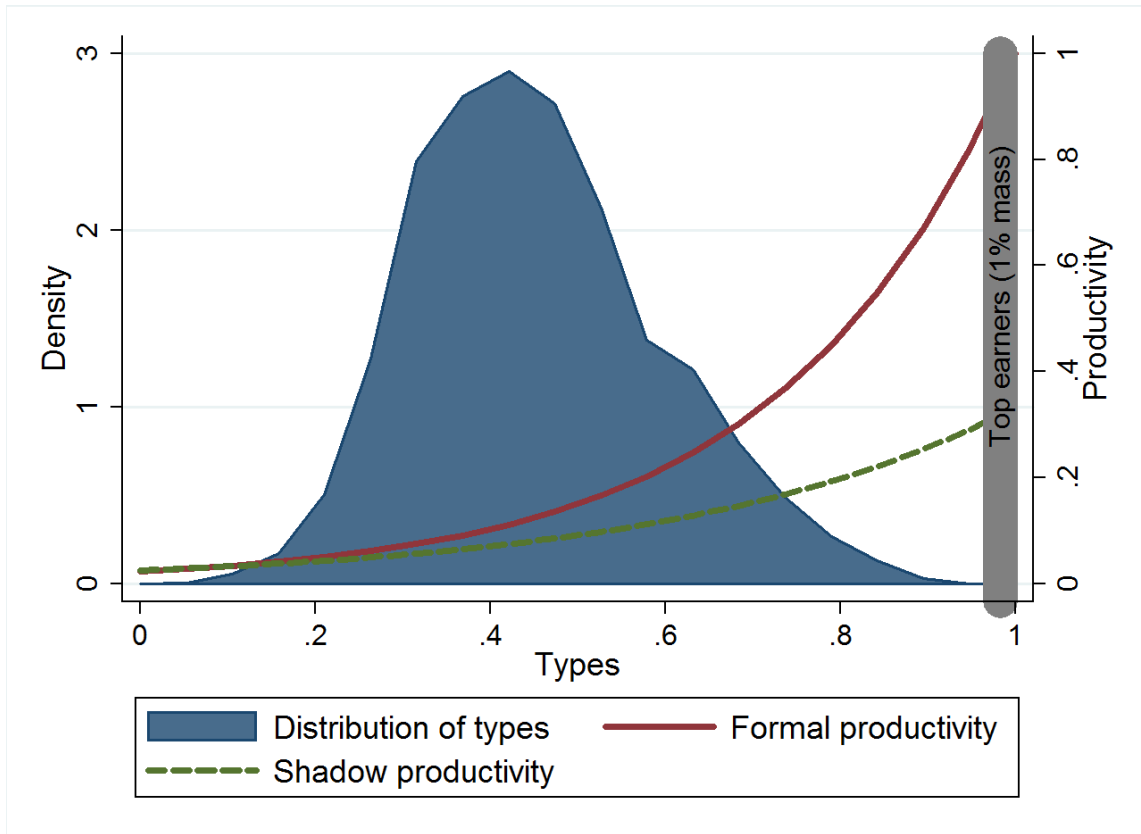
1.4.3 Estimation results

Here we discuss the results of the estimation of the formal productivity (w_i^f), the informal productivity (w_i^s) and the distribution of types (μ_i). Parameter estimates for β and the detailed description of the variables included in X_i are presented in Appendix 1.6.

Figure 1.8 presents the estimated productivities and the types distribution for non-top earners. The estimated values of γ_0^f and γ_0^s are almost identical with $\hat{\gamma}_0^s$ slightly greater so type 0 is slightly more productive in the shadow economy. The single-crossing condition is supported by the data since the hypothesis $\gamma_1^s < 1$ is not rejected at a 1% confidence level. The most productive individual among non-top earners is almost three times more productive in the formal economy than in the shadow economy.

Top earners are assigned to the set $[0.98, 1]$, the estimated value of the shape parameter of the Pareto distribution is 1.81 and comprise a mass of about 1% of the total population (details of the estimation are presented in Appendix 1.6). The shaded region in Figure 1.8 corresponds to the top earners. We do not plot their productivity profiles and density. Recall that what is identified is the distribution of formal productivities at the top with support $[\max_i \{\hat{w}_{it}^f\}, \infty)$ and this can be matched with many different combinations of formal productivity and probability density specifications in the types space; all of them equivalent for the optimal taxation problem that solves the planner. We assume that the relation between the shadow and the formal productivity from the main part of the distribution of types holds also for the top earners.

Figure 1.8: Estimated productivities and types distributions



1.5 Calibrated exercise

Given the productivity schedules estimated in the previous section, we calibrate the utility function and derive the optimal allocations for Colombia.

1.5.1 Calibration of the utility function

We assume that the agents' utility function is

$$U(c, n) = \log \left(c - \Gamma \frac{n^{1+\frac{1}{\zeta}}}{1 + \frac{1}{\zeta}} \right), \quad n \in [0, 1]. \quad (1.48)$$

The parameter ζ is the elasticity of labor supply. Since we consider a permanent tax reform, the relevant notion is the steady-state intensive margin elasticity. We fix ζ at different values and find Γ which minimizes the deviation of selected K model moments $(m_k^{model}(\zeta, \Gamma))_{k=1}^K$ from the corresponding data moments $(m_k^{data})_{k=1}^K$ according to the loss function

$$L(\zeta, \Gamma) = \sum_{k=1}^K \left(\frac{m_k^{model}(\zeta, \Gamma) - m_k^{data}}{m_k^{data}} \right)^2. \quad (1.49)$$

We use three moments: the share of shadow workers in total employment, the share of shadow income in total income and the average total income. The first two moments capture the relative size of the shadow economy, while the third one controls for the total production of Colombia. [Chetty, Guren, Manoli, and Weber \(2011\)](#) recommend using the steady-state intensive elasticity of 0.33, which we treat as a benchmark. However, the estimates behind this number implicitly incorporate responses on multiple margins, possibly also shifting labor to the shadow economy. Since we model this response explicitly, the correct value of elasticity could lower. Hence, we consider also the values of 0.2 and 0.1. Table 1.1 shows the matched moments for different values of the elasticity of labor supply.

Table 1.1: Calibration of the elasticity of labor supply

Moments	Actual economy	Model economy for different values of elasticity ζ		
		$\zeta = 0.33$	$\zeta = 0.2$	$\zeta = 0.1$
share of shadow workers	57.99%	64.51%	62.12%	60.53%
share of shadow income	30.94%	23.25%	25.24%	26.64%
mean total income [USD]	7166	6673	6659	6677

The model replicates well the magnitude of the shadow economy for a range of elasticities of labor supply. We conclude that the empirical distribution of productivities and the actual tax schedule can explain the high level of informality in Colombia.

1.5.2 Optimal allocations

We find the optimum for the two social welfare functions. First, we use the Rawlsian welfare criterion, which puts all the weight on the individual with the lowest utility level. Since both formal and shadow productivities are increasing with type, the Rawlsian planner cares only about the lowest type. Second, we derive the Utilitarian optimum with the planner that maximizes the average utility level in the economy. In each case we require that the planner obtains the same net tax revenue as the actual tax schedule.

The optimal allocations are described in Table 1.2. The Rawlsian planner would displace close to 22% of the workforce to informality. The share of shadow income falls even more, since only the least productive workers end up in the shadow economy. The Utilitarian planner would cut the size of the informal sector even more, to less than 1%. The Utilitarian planner cares mainly about workers in the middle of the distribution, where the density of types is high. Hence, this planner is not willing to set high marginal tax rates at the bottom, as it would reduce the utility of the workers in the middle. As the tax rate at the bottom is low, few workers are displaced to the shadow economy.

The welfare gains from implementing the optimum are large. The Rawlsian planner manages to increase the transfers to the workers with no formal income by 85% in comparison to the actual tax and transfer system. It translates into welfare gains of 40% to 50% in consumption equivalent terms. The Utilitarian planner takes into consideration the welfare cost of increased taxation of the high types and expands the redistribution less. Nevertheless, the transfers received by the bottom types increase by more than 55% in comparison to the actual tax system in Colombia and welfare gains are close to of 20% in terms of consumption. In order to make sure that the welfare gains are not driven by a thick Pareto tail at the top, we recompute the optima without the top tail (see the last row of Table 1.2).²³ The welfare gains are naturally smaller, since the top earners constitute a sizable source of tax revenue. However, it is clear that most of the welfare gains come from the efficient taxation of the ordinary workers and not from the very rich.

Figure 1.9 demonstrates how the optimal tax schedule is determined. Recall that the shadow economy imposes an upper bound on the tax rate. If the tax rate of type i exceeds $1 - w_i^s/w_i^f$, the return to shadow labor is strictly greater than the return to formal labor. No agent of type i would be willing to supply formal labor at such terms. As is evident from the figure, all bottom types face tax rate above the upper bound. Hence, they are bunched together at the zero formal income. From equation (1.37) we know that workers who are not bunched face the marginal tax rate that is a minimum of the two expressions: the standard Mirrleesian tax rate given by a [Diamond \(1998\)](#) and the upper bound $1 - w_i^s/w_i^f$. In all our calibrations the upper bound plays a dominant role (see Figure 1.9). For the Utilitarian planner with elasticity of 0.33 the standard Mirrleesian tax

²³In this case the distribution of types has finite support. The mass of the excluded tail is 0.0045.

OPTIMAL REDISTRIBUTION WITH A SHADOW ECONOMY

Table 1.2: Optimal allocations

Moments	Actual economy	Optimal Rawlsian allocation			Optimal Utilitarian allocation		
		$\zeta = 0.33$	$\zeta = 0.2$	$\zeta = 0.1$	$\zeta = 0.33$	$\zeta = 0.2$	$\zeta = 0.1$
share of shadow workers	57.99%	21.68%	21.68%	21.68%	0.17%	0.18%	0.19%
share of shadow income	30.94%	5.59%	6.33%	6.98%	0.02%	0.03%	0.03%
mean total income [USD]	7165	6671	6967	7112	6825	7086	7245
welfare (cons. equiv.)	100%	151.8%	147.8%	142%	121.3%	120.9%	119.7%
welfare w/o top tail (cons. equiv.)	100%	136.5%	135%	133.6%	116.8%	117%	117.4%

rate dives under the upper bound just for some high types. For the Rawlsian planner, as well as in the cases of lower elasticity of labor supply, the Mirrleesian tax rate does not intercept the upper bound below the upper tail and hence does not influence the optimal tax in the main part of distribution. In contrast, in all our calibrations some of the upper tail workers are taxed according to the [Diamond \(1998\)](#) formula (the upper tail is not represented on Figure 1.9). We conclude that the optimal tax schedule of workers below the upper tail is predominantly determined by the shadow economy considerations. However, the usual labor supply responses are important for taxing very productive workers.

Figure 1.9: The role of the upper bound

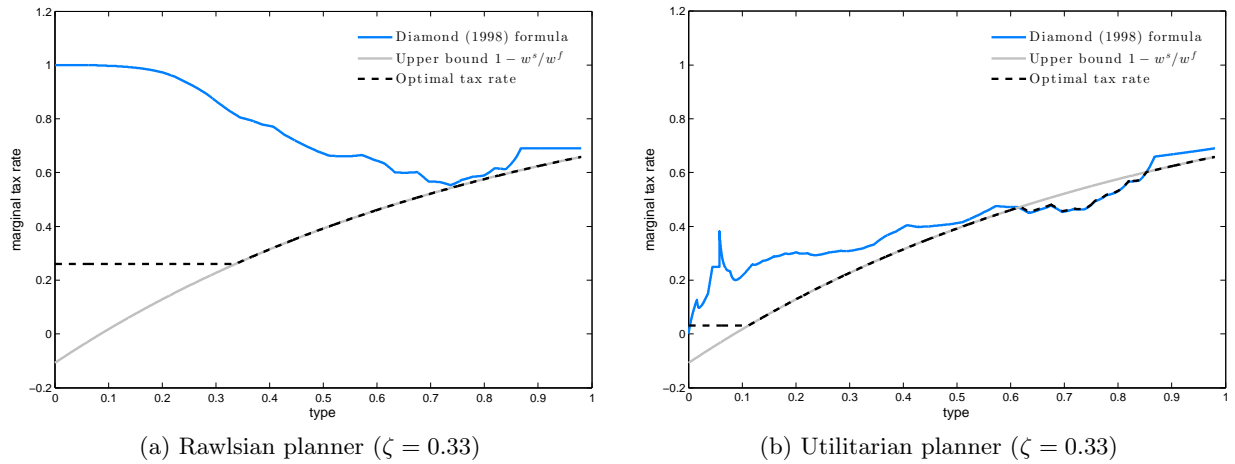
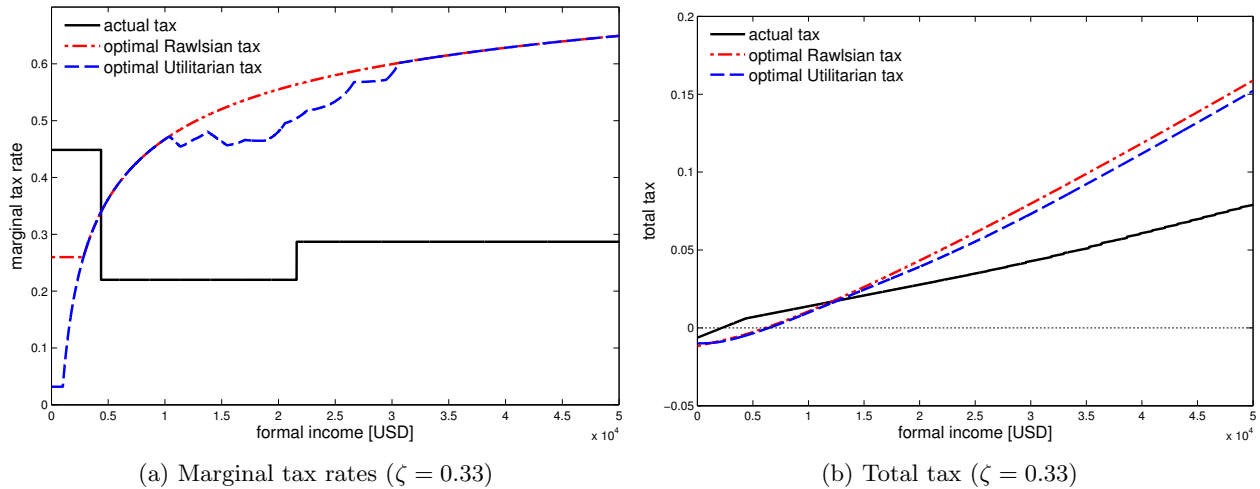


Figure 1.9 informs us also what would happen if the shadow economy was neglected and the standard Mirrleesian tax was implemented. All the types for which the tax rate exceeds the

upper bound would be displaced to the shadow economy. Moreover, many types for which the Mirrleesian tax rate is below the upper bound are likely to move to the shadows as well.²⁴ Hence, the implementation of the usual tax formula which does not account for the shadow economy would lead to a dramatic fall in tax revenue.

How does the optimal tax schedule compares with the one implemented at the time in Colombia? The actual tax schedule involves high 45% marginal rate at low levels of income, implied by phasing-out of transfers (see Figure 1.10). As income increases the rate drops to 22% and remains flat - workers with this income pay only the flat payroll tax. The progressive income tax starts at the high income level and gradually increases the marginal tax, reaching 49% for the top earners (at income levels not represented at Figure 1.10)).²⁵ In comparison to the actual tax rate, the optimal tax rates are lower at low levels of income and much higher elsewhere. Lower marginal rates at the bottom mean transfers are phased-out more slowly, so less productive workers have less incentives to move to the informal sector. Higher marginal tax rates elsewhere imply that the richest agents pay much higher total tax than in the actual economy, which allows the planner to finance the generous transfer (Figure 1.10 (b)). The tax rates at lower elasticities have very similar shape, as they are determined by the upper bound.

Figure 1.10: The optimal tax schedule



²⁴The tax burden accumulated at the low income levels is likely to outweigh the gain from higher return to formal labor at the high income levels.

²⁵The progressive tax is a step function with more than 80 steps of varying width and Figure 1.10 (a) shows its smoothly approximation. The true tax involves 0 rate at the interior of each step and an unbounded rate between steps, hence it cannot be represented on such graph.

1.6 Conclusions

A large fraction of the economic activity in most countries is informal. This paper incorporates this fact into the optimal income tax theory. We find that the shadow economy puts severe restrictions on the taxes the government can levy, often leading to a welfare loss. However, in some cases the shadow economy can raise welfare by improving both redistribution and efficiency. If the informal sector suppresses productivity differences between workers, the government can tax high earners more when the low productivity workers are employed informally. Furthermore, the shadow economy shelters poor workers from distortions implied by the taxation of the rich, allowing for more efficient allocation of labor.

The mechanism proposed has a quantitatively sizable effect. In the case of Colombia, the government that cares only about the poor would optimally choose to have 22% of workers in the shadow economy. Nevertheless, the observed levels of informality are much higher than that. According to our model, the large size of the Colombian shadow economy is explained by high marginal tax rates at low levels of income. The optimal tax schedule features lower rates at the bottom, leading to a smaller informal sector, and higher rates above, raising more revenue from top earners.

This paper suggests that allowing less productive people to collect welfare benefits and simultaneously work in the shadow economy could be desirable. Moreover, policies designed to deter the creation of informal jobs should focus on the jobs taken by the workers with the potential for high formal earnings. It is important to stress that the way the shadow economy is modeled in this paper abstracts from many issues, such as competition between formal and informal firms, lack of regulation and law enforcement, as well as potential negative externalities caused by the informal activity. All those phenomena are likely to reduce the potential welfare gains from exploiting the shadow economy.

Appendix

Proofs from Section 1.2

Proof of Proposition 1.1. Omitted. \square

Proof of Proposition 1.2. Note that the first-best allocation is consistent with the additional constraint (1.5), hence it is the solution to the planner's problem. Essentially, conditional on truthfully revealing type, incentives of the agent and the planner regarding the shadow labor are perfectly aligned. If a given type pays taxes according to the true type, choosing shadow labor in order to maximize utility cannot hurt the social welfare. \square

Proof of Proposition 1.3. In the first-best, $U(c_L, n_L) \geq U(c_H, n_H)$. By assumption of $v'(0) = 0$, we know that $n_L^f > 0$. Then the utility of H mimicking L is $U\left(c_L, \frac{w_L^f}{w_H} n_L^f\right) > U(c_L, n_L^f) \geq U(c_H, n_H)$, which violates $IC_{H,L}$. Hence, the optimum is not the first-best.

Suppose that at the optimum $IC_{H,L}$ does not bind. First, let's consider the case in which $U(c_H, n_H) > U(c_L, n_L)$. Since $IC_{H,L}$ is slack, the planner may increase transfers from H to L , which raises welfare, so it could not be the optimum in the first place. Second, suppose that $U(c_L, n_L) \geq U(c_H, n_H)$. It can happen only if $n_L^s > 0$. Otherwise, as we have shown above, $IC_{H,L}$ is violated. If $n_L^s > 0$ and $IC_{H,L}$ is slack, the planner can marginally decrease n_L^s and increase n_L^f , which generates free resources. Hence, at the optimum $IC_{H,L}$ has to bind.

Suppose that $IC_{L,H}$ binds. If the resource constraint is satisfied as equality, it may happen only if L type is paying a positive tax, while H type receives a transfer. Then the planner can improve welfare by canceling the redistribution altogether and reverting to *laissez-faire*, where none of the incentive constraints bind. \square

Lemma 1.3. *At the optimum either $U(c_L, n_L) = U(c_H, n_H)$ and $n_L^s > 0$, or the following optimality condition holds*

$$\min \left\{ \frac{v'(n_L)}{w_L^f} - \left(\mu_L + \mu_H \frac{v'(n_{H,L})}{w_H^f} \right), n_L^f \right\} = 0, \quad (1.50)$$

where $n_{i,-i} = \frac{w_{-i}^f}{w_i^f} n_{-i}^f + n_i^s \left(\frac{w_{-i}^f}{w_i^f} n_{-i}^f \right)$ is the total labor supply of type i pretending to be of type $-i$.

Suppose that v'' is nondecreasing. If $\frac{w_H^f}{w_L^f} g(w_H^s) \geq g(w_L^s)$ then this optimality condition is sufficient for the optimum.

Proof of Lemma 1.3. If $U(c_L, n_L) = U(c_H, n_H)$ and $n_L^s = 0$, then such allocation is not incentive compatible. The proof is identical as the proof of the claim that the first-best is not incentive compatible in Proposition 1.3. Hence, if $U(c_L, n_L) = U(c_H, n_H)$, then $n_L^s > 0$.

Let's consider the case in which $U(c_H, n_H)$ is always greater than $U(c_L, n_L)$. $IC_{H,L}$ has to bind, otherwise the planner could equalize utilities of both types. Consider changing n_L^f by a small amount and adjusting T_L such that $IC_{H,L}$ is satisfied as equality. It means that

$$\frac{dT_L}{dn_L^f} = w_L^f \mu_H \left(1 - \frac{v'(n_{H,L})}{w_H^f} \right).$$

This perturbation affects social welfare by

$$\frac{dU(c_L, n_L)}{dn_L^f} = w_L^f - \frac{\partial T_L}{\partial n_L^f} - v'(n_L) = w_L^f \left(\mu_L + \mu_H \frac{v'(n_{H,L})}{w_H^f} \right) - v'(n_L).$$

Optimum requires that either $\frac{dU(c_L, n_L)}{dn_L^f} = 0$ or $\frac{dU(c_L, n_L)}{dn_L^f} < 0$ and $n_L^f = 0$, which results in (1.50). Sufficiency of this first order condition depends on the second order derivative of welfare with respect to the perturbation. In order to have the second derivative well behaved, we are going to assume that v'' is nondecreasing. Then, we need to consider two cases (see Table 1.3). If $\frac{w_H^f}{w_L^f} g(w_H^s) \geq g(w_L^s)$ holds, then $\frac{dU(c_L, n_L)}{dn_L^f}$ is non-increasing in n_L^f . It means that the optimality condition (1.50) is sufficient. If $\frac{w_H^f}{w_L^f} g(w_H^s) < g(w_L^s)$, then $\frac{dU(c_L, n_L)}{dn_L^f}$ is not monotone in n_L^f and it may be the case that (1.50) points at either local maximum which is not a global maximum, or at the local minimum.

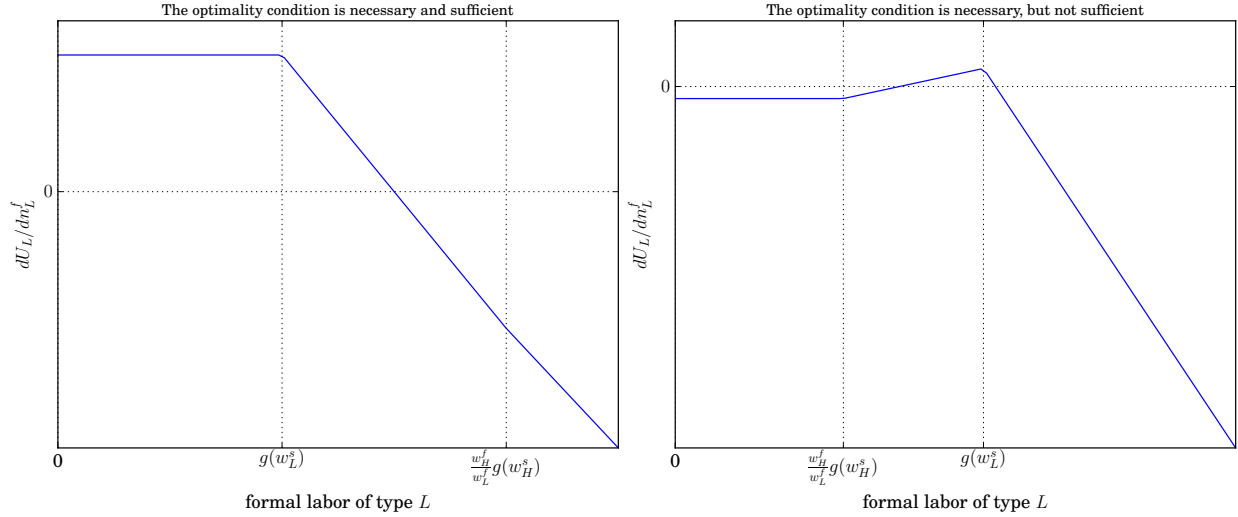
Figure 1.11 shows these two cases. In the first panel $\frac{w_H^f}{w_L^f} g(w_H^s) \geq g(w_L^s)$ holds and the optimality condition (1.50) always points at the optimum (in this case, the value of n_L^f where $\frac{dU(c_L, n_L)}{dn_L^f} = 0$).

In the second panel $\frac{w_H^f}{w_L^f} g(w_H^s) < g(w_L^s)$ holds and the optimality condition is not sufficient. There are three points that satisfy condition (1.50): local maximum at $n_L^f = 0$, local minimum with $n_L^f \in \left(\frac{w_H^f}{w_L^f} g(w_H^s), g(w_L^s) \right)$ and the other local maximum with $n_L^f > g(w_L^s)$. \square

Table 1.3: Second order derivative of welfare with respect to the perturbation

The case of $\frac{w_H^f}{w_L^f} g(w_H^s) \geq g(w_L^s)$			
	$n_L^f < g(w_L^s)$	$g(w_L^s) < n_L^f < \frac{w_H^f}{w_L^f} g(w_H^s)$	$\frac{w_H^f}{w_L^f} g(w_H^s) < n_L^f$
$\frac{d^2 U(c_L, n_L)}{dn_L^{f^2}} =$	0	$-v''(n_L^f) < 0$	$\mu_H \left(\frac{w_L^f}{w_H^f} \right)^2 v'' \left(\frac{w_L^f}{w_H^f} n_L^f \right) - v''(n_L^f) < 0$
The case of $\frac{w_H^f}{w_L^f} g(w_H^s) < g(w_L^s)$			
	$n_L^f < \frac{w_H^f}{w_L^f} g(w_H^s)$	$\frac{w_H^f}{w_L^f} g(w_H^s) < n_L^f < g(w_L^s)$	$g(w_L^s) < n_L^f$
$\frac{d^2 U(c_L, n_L)}{dn_L^{f^2}} =$	0	$\mu_H \left(\frac{w_L^f}{w_H^f} \right)^2 v'' \left(\frac{w_L^f}{w_H^f} n_L^f \right) > 0$	$\mu_H \left(\frac{w_L^f}{w_H^f} \right)^2 v'' \left(\frac{w_L^f}{w_H^f} n_L^f \right) - v''(n_L^f) < 0$

Figure 1.11: Sufficiency of the optimality condition



Proof of Proposition 1.4. In the proof of Lemma 1.3 above we described the impact of changing formal labor of L on the social welfare, $\frac{dU(c_L, n_L)}{dn_L^f}$. The condition (1.10) describes situations when the impact of the perturbation is non-positive at $n_L^f = 0$. From Figure 1.11 it is clear that if it is not the case, type L will never optimally work in the shadow economy.

Suppose that $\frac{w_H^f}{w_L^f} g(w_H^s) \geq g(w_L^s)$. Condition (1.10) implies that $\frac{dU(c_L, n_L)}{dn_L^f}$ is always non-positive, so it is optimal to reduce n_L^f as long as $U(c_H, n_H) > U(c_L, n_L)$. From Lemma 1.3 we know also that $U(c_H, n_H) > U(c_L, n_L)$ if L works only formally, so it is optimal to place type L in the shadow economy.

Now suppose that $\frac{w_H^f}{w_L^f} g(w_H^s) < g(w_L^s)$. Condition (1.11) means that the maximum of $\frac{dU(c_L, n_L)}{dn_L^f}$

attained at $n_L^f = g(w_L^s)$ (see Figure 1.11) is non-positive. Therefore, it is optimal to reduce n_L^f until utilities of both types are equalized, which can happen only when L works in the shadow economy. Condition (1.11) is sufficient, but not necessary for L to work in the shadow economy, because the social welfare changes in a non-monotone way with n_L^f . If (1.11) is not satisfied, marginally increasing n_L^s from 0 is bad for welfare, but increasing it further may eventually lead to welfare gains, and the total effect on welfare is ambiguous. \square

Proof of Proposition 1.5. Suppose that optimally $n_L^s > 0$. From Figure 1.11 it is clear that in such situation it is in the best interest of type L to work exclusively in the shadow economy. However, if $w_L^s > w_H^s$ and $n_L^f = 0$, the incentive compatibility constraint of the type H implies that

$$U(c_L, n_L) = U(w_L^s n_L^s - T_L, n_L^s) > U(w_H^s n_{H,L}^s - T_L, n_{H,L}^s) = U(c_H, n_H).$$

Since the planner is Rawlsian, such allocation is not desirable. The planner will rather stop decreasing n_L^f at the point where utilities of both types are equal. On the other hand, if $w_L^s \leq w_H^s$ then

$$U(c_L, n_L) = U(w_L^s n_L^s - T_L, n_L^s) \leq U(w_H^s n_{H,L}^s - T_L, n_{H,L}^s) = U(c_H, n_H),$$

so the planner will optimally decrease n_L^f to zero. \square

Proof of Proposition 1.6. In order to examine when the optimum welfare is strictly higher than in the standard Mirrlees model, we will compare utility of type L in the standard Mirrlees model ($U(c_L^M, n_L^M)$) and in the shadow economy model, when L is working only in the shadow economy ($U(c_L^{SE}, n_L^{SE})$). Clearly, when the second scenario yields higher utility, the existence of the shadow economy is welfare improving.

In the standard Mirrlees model, the binding constraint is $U(w_H^f n_H^M, n_H^M) - T_H^M = U(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M) - T_L^M$. Together with the resource constraint it means that $T_L^M = \mu_H \left(U(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M) - U(w_H^f n_H^M, n_H^M) \right)$. Now, the utility of type L is

$$U(c_L^M, n_L^M) = U(w_L^f n_L^M, n_L^M) - T_L = U(w_L^f n_L^M, n_L^M) - \mu_H \left(U(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M) - U(w_H^f n_H^M, n_H^M) \right).$$

Using the same steps, we can express the utility of type L working only in the shadow economy as

$$U(c_L^{SE}, n_L^{SE}) = U(w_L^s n_L^{SE}, n_L^{SE}) - \mu_H \left(U(w_H^s n_{H,L}^{SE}, n_{H,L}^{SE}) - U(w_H^f n_H^{SE}, n_H^{SE}) \right).$$

Since there are no distortions at the top and no wealth effects, $n_H^M = n_H^{SE}$. The shadow economy is

welfare improving, $U(c_L^{SE}, n_L^{SE}) - U(c_L^M, n_L^M) > 0$, iff

$$U(w_L^s n_L^{SE}, n_L^{SE}) - U(w_L^f n_L^M, n_L^M) + \mu_H \left(U\left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M\right) - U(w_H^s n_{H,L}^{SE}, n_{H,L}^{SE}) \right) > 0.$$

The first difference (the efficiency gain) is positive if $w_L^s > \bar{w}_L^s$. The second difference (the redistribution gain) is positive when $w_H^s < \bar{w}_H^s$. Hence, when both inequalities hold weakly and at least one holds strictly, the existence of the shadow economy improves welfare in comparison to the standard Mirrlees model.

Now we will show that when the inequalities hold in the other direction, the shadow economy hurts welfare. Suppose that $w_L^s = \bar{w}_L^s$ and $w_H^s = \bar{w}_H^s$. We will prove that allocation SE is a unique optimum at this point. First we will show that when the redistribution gain is non-positive, it is true that $n_H^{SE} > \frac{w_L^f}{w_H^f} n_L^M$. Suppose on the contrary that $n_H^{SE} \leq \frac{w_L^f}{w_H^f} n_L^M$. Then we can write the following sequence of inequalities

$$U\left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M\right) \geq U\left(w_H^f n_H^{SE}, n_H^{SE}\right) > U\left(w_H^s n_H^{SE}, n_H^{SE}\right).$$

The first inequality comes from the fact that $\frac{w_L^f}{w_H^f} n_L^M$ is below the efficient level of labor supply of type H , so lowering the labor of this type even further to n_H^{SE} will decrease the utility. The second inequality is simply implied by our assumption $w_H^f > w_H^s$. This sequence of inequalities implies that the redistribution gain is strictly positive. Hence, if the redistribution gain is non-positive, $n_H^{SE} > \frac{w_L^f}{w_H^f} n_L^M$ holds.

Note that $n_H^{SE} > \frac{w_L^f}{w_H^f} n_L^M$ means that the optimal allocation of the standard Mirrlees model is not incentive-compatible with the shadow economy - deviating type H would supply some additional shadow labor. Hence, any allocation which yields the social welfare equal or higher than $U(c_L^M, n_L^M)$ has to involve type L working in the shadow economy.

Let's go back to the optimal allocation with the shadow economy, when $w_L^s = \bar{w}_L^s$ and $w_H^s = \bar{w}_H^s$. From the considerations above we know that the optimum involves some shadow labor. If we sum the efficiency gain and the redistribution gain divided by μ_H and rearrange the terms, we get

$$\left(U\left(w_L^f n_L^M, \frac{w_L^f}{w_H^f} n_L^M\right) - U\left(w_L^f n_L^M, n_L^M\right) \right) - (U(w_H^s n_H^{SE}, n_H^{SE}) - U(w_L^s n_L^{SE}, n_L^{SE})) = 0.$$

The expression in the first brackets is positive. Hence, the second brackets are positive as well, which means that $w_H^s > w_L^s$. By Proposition 1.5 type L will work exclusively in the shadow

economy.

To sum up, we know that at $(w_L^s, w_H^s) = (\bar{w}_L^s, \bar{w}_H^s)$ the optimum of the shadow economy model is unique and involves type L working entirely in the shadow economy. Consequently, a decrease in the shadow productivity of type L or an increase in the shadow productivity of type H leads to a strict welfare loss, since it either decreases the effective productivity of type L or decreases the transfer type L receives. \square

Proof of Proposition 1.7. Suppose that $\lambda_i \leq \lambda_{-i}$. In this case the $IC_{i,-i}$ may bind (it will if the inequality is strict), while $IC_{-i,i}$ is always slack. The planner will not distort the allocation of type i . Without distortions, this type will never work in the shadow economy.

Suppose that $\lambda_i > \lambda_{-i}$, so that $IC_{-i,i}$ binds. Perturb n_i^f and adjust T_i such that $IC_{-i,i}$ holds as equality:

$$\frac{dT_i}{dn_i^f} = w_i^f \mu_{-i} \left(1 - \frac{v'(n_{-i,i})}{w_{-i}^f} \right).$$

This perturbation affects social welfare by

$$\begin{aligned} \frac{dW}{dn_i^f} &= \lambda_i \mu_i \left(w_i^f - \frac{\partial T_i}{\partial n_i^f} - v'(n_i) \right) + \lambda_{-i} \mu_{-i} \frac{\mu_i}{\mu_{-i}} \frac{\partial T_i}{\partial n_i^f} \\ &= \lambda_i \mu_i w_i^f \left(\left(1 - \frac{v'(n_i)}{w_i^f} \right) + \left(\frac{\lambda_{-i}}{\lambda_i} - 1 \right) \mu_{-i} \left(1 - \frac{v'(n_{-i,i})}{w_{-i}^f} \right) \right) \end{aligned} \quad (1.51)$$

Suppose that $\frac{w_{-i}^s}{w_{-i}^f} \geq \frac{w_i^s}{w_i^f}$ and $n_i^f \leq g(w_i^s)$, which means that $v'(n_i) = w_i^s$. Note that $\frac{v'(n_{-i,i})}{w_{-i}^f} \geq \frac{w_{-i}^s}{w_{-i}^f} \geq \frac{w_i^s}{w_i^f}$. Hence

$$1 - \frac{w_i^s}{w_i^f} \geq 1 - \frac{v'(n_{-i,i})}{w_{-i}^f} > \left(1 - \frac{\lambda_{-i}}{\lambda_i} \right) \mu_{-i} \left(1 - \frac{v'(n_{-i,i})}{w_{-i}^f} \right),$$

which means that $\frac{dW}{dn_i^f} > 0$. Therefore, it is never optimal to decrease n_i^f so much that type i works in the shadow economy. \square

Proof of Proposition 1.8. First we will show how to obtain (1.16). The efficiency gain is straightforward. In order to obtain the redistribution gain, note that there are no distortions imposed on type $-i$, hence

$$\mu_{-i} \lambda_{-i} (U(c_{-i}^{SE}, n_{-i}^{SE}) - U(c_{-i}^M, n_{-i}^M)) = \mu_{-i} \lambda_{-i} (T_{-i}^M - T_{-i}^{SE}) = -\mu_i \lambda_{-i} (T_i^M - T_i^{SE}).$$

Summing up the terms results in (1.16). In order to derive thresholds, recall that $H(w^s) = U(w^s g(w^s), g(w^s))$. The efficiency gain is given by

$$\mu_i \lambda_i \left(H(w_i^s) - U(w_i^f n_i^M, n_i^M) \right),$$

it is strictly increasing in w_i^s and positive for $w_i^s > \bar{w}^s_i$. Note that by (1.51) n_i^M will always be distorted (downwards if $i = l$, upwards if $i = h$). Hence, $U(w_i^f n_i^M, n_i^M) < H(w_i^f)$ and the threshold \bar{w}^s_i is strictly lower than w_i^f .

Using the binding $IC_{-i,i}$ constraint, we can express the redistribution gain as

$$\mu_i \mu_{-i} (\lambda_i - \lambda_{-i}) \left(U\left(w_i^f n_i^M, \frac{w_i^f}{w_{-i}^f} n_i^M\right) - H(w_{-i}^s) \right).$$

It is strictly decreasing in w_{-i}^s and is positive for $w_{-i}^s < \bar{w}^s_{-i}$. Since $\frac{w_i^f}{w_{-i}^f} n_i^M \neq g(w_{-i}^f)$, it is true that $U\left(w_i^f n_i^M, \frac{w_i^f}{w_{-i}^f} n_i^M\right) < H(w_{-i}^f)$ and the threshold \bar{w}^s_{-i} is strictly lower than w_{-i}^f . \square

Proofs from Section 1.3

Proof of Lemma 1.1. The single-crossing requires that $\frac{d}{di} \left(\frac{\partial V_i(y^f, T)/\partial y^f}{\partial V_i(y^f, T)/\partial T} \right) < 0$. Suppose that $v'\left(\frac{y^f}{\phi_i}\right) < \psi_i$. Then the agent supplies no informal labor and the indirect utility function V is just the utility function U evaluated at the formal allocation. Since v' is increasing, the single crossing holds in this case. When $v'\left(\frac{y^f}{\phi_i}\right) \geq \psi_i$, then the optimal provision of informal labor means that $v'(n_i) = w_i^f$, which implies $\frac{\partial V_i(y^f, T)/\partial y^f}{\partial V_i(y^f, T)/\partial T} = \frac{w_i^s}{w_i^f}$. Therefore the single crossing condition requires that $\frac{d}{di} \left(\frac{w_i^s}{w_i^f} \right) < 0$. \square

Proof of Proposition 1.9. First note that the incentive compatibility requires that if $\frac{d}{dj} V_i(y_j^f, T_j) \Big|_{j=i}$ exists, it is equal to 0. Otherwise type i can improve welfare by changing income marginally, so the allocation is not incentive compatible. Hence, if $\frac{d}{di} V_i(y_i^f, T_i) = \frac{d}{dj} V_i(y_j^f, T_j) \Big|_{j=i} + \frac{d}{di} V_i(y_j^f, T_j) \Big|_{j=i}$ exists, it is equal to $\frac{d}{di} V_i(y_j^f, T_j) \Big|_{j=i} = \left(\frac{\dot{w}_i^f}{w_i^f} n_i^f + \frac{\dot{w}_i^s}{w_i^s} n_i^s \right) v'(n_i)$. We call this derivative a marginal information rent and denote it simply by \dot{V}_i .

By [Milgrom and Segal \(2002\)](#) (see their 10th footnote and Theorem 2), we can represent the utility

schedule for any $i < 1$ as an integral of marginal information rents

$$V_i(y_i^f, T_i) = V_0(y_0^f, T_0) + \int_0^i \dot{V}_j dj,$$

Moreover, the utility schedule V_i is continuous everywhere and differentiable almost everywhere.

Now we will show that the allocation is not incentive compatible if the formal income is decreasing in type. Suppose that the allocation is incentive-compatible and that there are two types $a < b$ such that $y_a^f > y_b^f$. By the incentive compatibility, we have

$$V_a(y_a^f, T_a) \geq V_a(y_b^f, T_b). \quad (1.52)$$

$\frac{d}{di} V_i(y^f, T)$ is increasing in y^f . To see it, note that

$$\frac{d}{di} V_i(y^f, T) = \left(\rho_i^f \frac{y^f}{w_i^f} + \rho_i^s \max \left\{ g(w_i^s) - \frac{y^f}{w_i^f}, 0 \right\} \right) v'(n_i),$$

where g is the inverse function of v' . The single-crossing implies that $\rho_i^f > \rho_i^s$, so the right hand side is increasing in y^f .

Since $y_a^f > y_b^f$, for each type i it is true that $\frac{d}{di} V_i(y_a^f, T_a) > \frac{d}{di} V_i(y_b^f, T_b)$. It implies

$$V_b(y_a^f, T_a) - V_a(y_a^f, T_a) = \int_a^b \frac{d}{di} V_i(y_a^f, T_a) di > \int_a^b \frac{d}{di} V_i(y_b^f, T_b) di = V_b(y_b^f, T_b) - V_a(y_b^f, T_b). \quad (1.53)$$

Summing (1.52) and (1.53) results in

$$V_b(y_a^f, T_a) > V_b(y_b^f, T_b),$$

which contradicts the incentive-compatibility. Therefore, a nondecreasing formal income schedule is necessary for incentive compatibility. Conversely, suppose that the local incentive constraints hold and the formal income schedule is nondecreasing. Then for any two types $a < b < 1$

$$V_b(y_b^f, T_b) - V_a(y_a^f, T_a) = \int_a^b \frac{d}{di} V_i(y_i^f, T_i) di \geq \int_a^b \frac{d}{di} V_i(y_a^f, T_a) di = V_b(y_a^f, T_a) - V_a(y_a^f, T_a), \quad (1.54)$$

which implies

$$V_b(y_b^f, T_b) \geq V_b(y_a^f, T_a).$$

We can use the same reasoning, but bound the utility difference on the left hand side of (1.54) from above by $\int_a^b \frac{d}{di} V_i(y_b^f, T_b) di$ to get

$$V_a(y_a^f, T_a) \geq V_a(y_b^f, T_b).$$

We cannot use this argument when $b = 1$ and w_1^f is unbounded, but then by continuity of V_i we have $\lim_{b \rightarrow 1} \left\{ V_b(y_b^f, T_b) - V_b(y_a^f, T_a) \right\} \geq 0$. \square

Proof of Theorem 1.1. First we will derive D_i^f and D_i^s term. Then we will show that conditions from the theorem are necessary. Finally we will prove sufficiency.

Suppose that $i \in \mathcal{F}$. A perturbation of formal income changes the marginal information rent of type i by

$$\frac{\partial \dot{V}_i}{\partial y_i^f} = (1 - t_i) \rho_i^f \left(1 + \frac{1}{\zeta_i} \right). \quad (1.55)$$

where $\zeta_i = \frac{v'(n_i)}{n_i v''(n_i)}$ is the elasticity of labor supply. This change of income affects the utility level of type i by $\frac{dV_i}{dy_i^f} = 1 - \frac{v'(n_i)}{w_i^f}$. By Proposition 1.9 the utility schedule has to be continuous, so we have to introduce additional change in tax T_i in order to keep the utility level of type i constant. We adjust the total tax paid by an agent of type i by $dT_i = 1 - \frac{v'(n_i)}{w_i^f}$. Note that dT_i is just equal the marginal tax rate t_i . This perturbation influences the tax revenue as if we were decreasing the formal income of type i while keeping the marginal tax rate fixed. Since we are interested in the tax revenue impact of the unit perturbation of the marginal information rent, we normalize dT_i by $\frac{\partial \dot{V}_i}{\partial y_i^f}$. In order to capture the tax revenue impact of perturbation of all agents of type i , we multiply this expression by μ_i and get

$$D_i^f = dT_i \left(\frac{\partial \dot{V}_i}{\partial y_i^f} \right)^{-1} \mu_i = \frac{t_i}{1 - t_i} \left(\rho_i^f \left(1 + \frac{1}{\zeta_i} \right) \right)^{-1} \mu_i.$$

Suppose that $i \in \mathcal{S}$. Shadow labor is supplied according to $v'(n_i) = w_i^s \implies n_i = g(w_i^s)$. The marginal information rent can be expressed as

$$\dot{V}_i = \left(\frac{\dot{w}_i^f}{(w_i^f)^2} y_i^f + \frac{\dot{w}_i^s}{w_i^s} \left(g(w_i^s) - \frac{y_i^f}{w_i^f} \right) \right) w_i^s. \quad (1.56)$$

We marginally perturb y_i^f . The impact of the perturbation of the marginal information rent is

$$\frac{d\dot{V}_i}{dy_i^f} = \left(\rho_i^f - \rho_i^s\right) \frac{w_i^s}{w_i^f}.$$

As in the formal workers' case, the perturbation of y_i^f alone changes the utility level of type i . In order to keep the utility schedule continuous at i , we need to adjust the tax T_i such that the utility of this type is unchanged. The required change of the tax is $dT_i = 1 - \frac{v'(n_i)}{w_i^f}$, which for the shadow worker equals $\frac{w_i^f - w_i^s}{w_i^f}$. By multiplying the tax revenue change with μ_i and normalizing it with $\frac{d\dot{V}_i}{dy_i^f}$, we obtain the tax revenue cost of decreasing the marginal information rent of type i :

$$D_i^s = dT_i \left(\frac{\partial \dot{V}_i}{\partial y_i^f} \right)^{-1} \mu_i = \frac{w_i^f - w_i^s}{w_i^s} \left(\rho_i^f - \rho_i^s \right)^{-1} \mu_i.$$

If the interior formal income is nondecreasing, the interior allocation implied is incentive-compatible. The necessity of the conditions (1.30)-(1.33) was demonstrated in the main text. If these conditions do not hold, there exists a beneficial perturbation.

The conditions (1.30)-(1.33) are sufficient when they pin down the unique interior allocation. This happens when the cost of distortions is decreasing in the formal income of each type. Then the government's problem of choosing formal income of each type is concave. For formal workers it requires that ζ_i is non-increasing in the labor supply, as then increasing the marginal tax rate t_i leads to an increase in D_i^f . For the marginal workers we need $D_i^s > D_i^f$, which is guaranteed by $\frac{\rho_i^s}{\rho_i^f} > \zeta_i^{-1}$. See the footnote 11 for the comment regarding the uniqueness of allocation for types for which $D_i^s = \int_i^1 N_j dj$ holds. \square

Proof of Proposition 1.10. We will examine the monotonicity of an interior formal income schedule separately for the formal, marginal and shadow workers.

The single-crossing condition implies that if the marginal tax rate is non-increasing in type, the formal income of workers in \mathcal{F} is increasing. By (1.30) the marginal tax rate satisfies

$$\frac{t_i}{1 - t_i} = \rho_i^f \left(1 + \frac{1}{\zeta_i} \right) \frac{1 - M_i}{\mu_i} \mathbb{E}(1 - \omega_j | j > i). \quad (1.57)$$

Assumption 1.3(i) means that $\mathbb{E}(1 - \omega_j | j > i) = \mathbb{E}\left(1 - \frac{\lambda_j}{\eta} \middle| j > i\right)$ is non-increasing in i . Assumptions 1.3(ii) and 1.3(iii) imply that the rest of the right hand side of (1.57) is non-increasing in i . Hence, t_i is non-increasing and the interior formal income schedule is increasing in \mathcal{F} .

For any marginal worker i the formal income is fixed at $w_i^f g(w_i^s)$. The derivative of formal income with respect to type is

$$\dot{y}_i^f = \frac{dw_i^f g(w_i^s)}{di} = \dot{w}_i^f g(w_i^s) + w_i^f \dot{w}_i^s g'(w_i^s) = w_i^f g(w_i^s) \left(\rho_i^f + \rho_i^s \frac{w_i^s g'(w_i^s)}{g(w_i^s)} \right).$$

Notice that $\frac{w_i^s g'(w_i^s)}{g(w_i^s)} = \frac{v'(n_i)}{n_i v''(n_i)} = \zeta_i$. Hence, for any marginal worker $\dot{y}_i^f \geq 0$ if and only if $\rho_i^f + \rho_i^s \zeta_i \geq 0 \Leftrightarrow \frac{\rho_i^s}{\rho_i^f} \geq -\zeta_i^{-1}$.

In the interior allocation all shadow workers have zero formal income. Hence, the formal income schedule is non-decreasing only if shadow workers are present exclusively at the bottom of the type space. According to (1.32), a worker i belongs to \mathcal{S} in an interior allocation if and only if

$$\frac{w_i^f - w_i^s}{w_i^s} \leq \rho_i^f \frac{1 - M_i}{\mu_i} \left(1 - \frac{\rho_i^s}{\rho_i^f} \right) \mathbb{E}(1 - \omega_j | j > i).$$

The left hand side is increasing in i by the single-crossing assumption. The right hand side is non-increasing by assumptions 1.3(i), 1.3(ii) and 1.3(iv). \square

Proof of Proposition 1.2. We will show that under the assumptions made the interior allocation is such that bottom types do not work in the shadow economy, while some types above them do. This leads to the income schedule locally decreasing in type.

Let's compute the term $\int_i^1 N_j dj$. By (1.33) we know that $\eta = \mathbb{E}\{\lambda_i\} = 1$. Hence $\int_i^1 N_j dj = \int_i^1 (1 - \lambda_j) dj$ and the derivative of this term is $\frac{\partial \int_i^1 N_j dj}{\partial i} = \lambda_i - 1$.

The term D_i^s is

$$D_i^s = \left(\frac{w_0^f}{w_0^s} e^{(\rho^f - \rho^s)i} - 1 \right) (\rho^f - \rho^s)^{-1}.$$

By (1.32) any type i is a shadow worker in the interior allocation if and only if $\int_i^1 N_j dj \geq D_i^s$. We can rewrite this inequality as

$$w_0^s \geq \frac{e^{(\rho^f - \rho^s)i}}{1 + (\rho^f - \rho^s) \int_i^1 N_j dj} w_0^f.$$

Denote the right hand side by X_i . Note that $X_0 = 1$, which together with $w_0^f > w_0^s$ implies that the bottom types do not work in the shadow economy and by the Assumption 1(iii) have a positive formal income.

We define the threshold \bar{w}_0^s as $\min_{i \in [0,1]} X_i$. In order to see that $\bar{w}_0^s < w_0^f$, let's compute the derivative of X_i :

$$\dot{X}_i = (\rho^f - \rho^s) e^{(\rho^f - \rho^s)i} \left(2 - \lambda_i + (\rho^f - \rho^s) \int_i^1 N_j dj \right).$$

Note that $\dot{X}_0 = (\rho^f - \rho^s) (2 - \lambda_0) < 0$, so X_i is decreasing at the bottom type and $\min_{i \in [0,1]} X_i < X_0 = w_0^f$. Therefore, whenever $w_0^f > w_0^s > \bar{w}_0^s$, the bottom types have a positive formal income, while some types above them work in the shadow economy and have no formal income. \square

Proof of Theorem 1.2.

Proof. There are three cases we should consider, depending on whether the interior formal income is increasing, locally decreasing, or increasing but not strictly. In the first case (strictly increasing schedule) by Theorem 1.1 the interior allocation is optimal. In the second case (locally decreasing schedule) by Theorem 1.2 we need to use the optimal bunching condition (1.35). Below we derive this condition formally. In the third case the interior income schedule is non-decreasing with flat parts. By Theorem 1.1 the interior allocation is optimal. However, Theorem 1.2 says that the flat parts of the income schedule should be consistent with the optimal bunching condition (1.35). We will show that those two approaches are equivalent.

Suppose that the formal income schedule y^f is constant at the segment of types $[a, b]$. Let's marginally decrease the formal income of types $[a, b)$. Since we don't change the allocation of types below a , we have to make sure that V_a is unchanged - otherwise the utility schedule becomes discontinuous. Together with the cut of the formal income, we have to introduce a change in the total tax paid at this income level $dT_a = 1 - \frac{v'(n_i)}{w_i^f} = t_{a-}$. Since all types $[a, b)$ are affected, the tax revenue loss is equal to

$$t_a (M_b - M_a). \quad (1.58)$$

Although this perturbation does not affect the utility of type a , it does affect the utility of all other bunched types. The utility impact of the perturbation of some type $i \in (a, b)$ equals $dU_i = 1 - \frac{v'(n_i)}{w_i^f} - dT_a = \frac{v'(n_a)}{\phi_a} - \frac{v'(n_i)}{\phi_i}$. The welfare loss of bunched agents due to this utility change is

$$\int_a^b \Delta MRS_i \omega_i d\mu, \quad (1.59)$$

where $\Delta MRS_i = \frac{v'(n_a)}{\phi_a} - \frac{v'(n_i)}{\phi_i}$. Having the fiscal and welfare loss at the kink, we can add them into a cost of increasing distortions at the bunch $[a, b)$. We normalize the sum by $t_{b+} - t_{a-}$, which makes sure that the perturbation results in the unit change of the utility of type b , and we obtain

(1.34). As the perturbation results in a uniform utility change of agents above the bunch, we can use the standard term (1.27) in order to obtain the optimal bunching condition (1.35).

Suppose that the interior formal income schedule is flat on the segment $[a, b]$. We will prove the equivalence of the interior optimality conditions and the optimal bunching condition. Let's consider the following sequence of perturbations. First, decrease the marginal information rent of agent a such that the formal income of this type falls by a unit. Take a marginally higher type and again perturb the marginal information rent such that the formal income of this agents is decreased by a unit as well. Do it until you reach type b . Note that incentive compatibility is preserved at each stage, since the formal income is always non-decreasing. The aggregate welfare impact of this sequence of perturbations is

$$W_{interior} = \int_a^b \frac{\partial \dot{V}_i}{\partial y} \left(D_i - \int_i^1 N_j dj \right) di,$$

where $D_i \equiv \begin{cases} D_i^f & \text{if } i \in \mathcal{F} \\ D_i^s & \text{if } i \in \mathcal{S} \end{cases}$. We do not need to consider the marginal workers, because their formal income is increasing (see the proof of Proposition 1.10), hence they cannot be bunched. We can decompose $W_{interior}$ into three components

$$W_{interior} = \underbrace{\int_a^b \frac{\partial \dot{V}_i}{\partial y} D_i di}_{X_1} - \underbrace{\int_a^b \frac{\partial \dot{V}_i}{\partial y} \int_i^b N_j dj di}_{X_2} - \underbrace{\int_a^b \frac{\partial \dot{V}_i}{\partial y} \int_b^1 N_j dj di}_{X_3}.$$

Note that $D_i = \frac{1 - MRS_i}{\frac{\partial \dot{V}_i}{\partial y}} \mu_i$, hence $X_1 = \int_a^c (1 - MRS_i) \mu_i di$. We observe that $\frac{\partial \dot{V}_i}{\partial y} = \frac{\partial^2 V_i}{\partial i \partial y} = -MRS_i$ and we integrate X_2 by parts

$$X_2 = - \int_a^b MRS_i \int_i^b N_j dj di = - \left(\left[MRS_i \int_i^b N_j dj \right]_a^b + \int_a^b MRS_i N_i di \right) = - \int_a^b (MRS_i - MRS_a) N_i di.$$

We simply integrate X_3

$$X_3 = - \int_a^b MRS_i \int_b^1 N_j dj di = - (MRS_b - MRS_a) \int_b^1 N_j dj.$$

Now by summing and rearranging the terms we get

$$\begin{aligned}
 W_{interior} &= X_1 - X_2 - X_3 \\
 &= \int_a^b (1 - MRS_i) d\mu + \int_a^b (MRS_i - MRS_a) (1 - \omega_i) d\mu + (MRS_b - MRS_a) \int_b^1 N_j dj \\
 &= \int_a^b (1 - MRS_a) d\mu + \int_a^b (MRS_a - MRS_i) \omega_i d\mu + (MRS_b - MRS_a) \int_b^1 N_j dj \\
 &= t_{a-} (M_b - M_a) + \int_a^b \Delta MRS_i \omega_i d\mu + (t_{a-} - t_{b+}) \int_b^1 N_j dj = (t_{b+} - t_{a-}) \left(D_{a,b} - \int_b^1 N_j dj \right).
 \end{aligned}$$

Since $t_{b+} - t_{a-} = \frac{v'(n_a)}{\phi_a} - \frac{v'(n_b)}{\phi_b} > 0$, the sequence of interior optimality conditions is equivalent to the optimal bunching condition (1.35). \square

Proof of Proposition 1.11. If the interior allocation is incentive-compatible, the claim holds. Suppose that it is not the case, i.e. there is a kink in the tax schedule. In this case incentive compatibility constrains the government from reducing the utility of agents above kink as much as in the interior case. Since G is concave, it means that N_j terms for j above the kink is weakly higher and the government's will to impose distortions does not decrease. If there are shadow workers at the bottom and the curve $\int_i^1 N_j dj$ shifts upwards, then even more types will be bunched at zero formal income at the bottom.

Let's think about shadow workers which are not at the bottom of the type space. The continuity assumptions guarantee that D_i^s and $\int_i^1 N_j dj$ terms are continuous in type. It implies that before any set of shadow workers that are not at the bottom of the type space is a marginal worker. Consider an interior allocation with a bunch of shadow workers at some positive formal income level. If we flatten the interior formal income schedule in order to make it non-decreasing (as in

Figure 1.6), the first type in the bunch (type \bar{a}) will be a marginal worker ($\frac{v'(\frac{y_{\bar{a}}}{w_{\bar{a}}^f}}{w_{\bar{a}}^s} = 1$), while

all the other types with this level of formal income will be shadow workers ($\frac{v'(\frac{y_{\bar{a}}}{w_i^f}}{w_i^s} < 1, i > \bar{a}$).

To see this, note that $\frac{\partial}{\partial_i} \left(\frac{v'(\frac{y_{\bar{a}}}{w_i^f})}{w_i^s} \right)$ is negative by $\frac{\rho_i^s}{\rho_f^s} > -\zeta^{-1}$. So far we discussed what happens at the flattened income schedule. The optimal income schedule involves no less distortions, so the shadow workers will not cease to supply shadow labor. \square

Proof of Corollary 2. It is just an interior optimality condition for the shadow worker (1.32). By Lemma 1.11, all the shadow workers from the interior allocation are shadow workers in the optimum. \square

The estimation of the factor F_i and top earners Pareto distribution.

Here we present the variables included in the vector X_i and the parameter estimates of β and γ obtained from the specification given by (1.44). Table 1..4 lists the variables included in X_i with its corresponding description and associated category. The parameter estimates are presented in Table 1..5. Finally, table 1..6 presents the estimate of the Pareto distribution for top earners.

OPTIMAL REDISTRIBUTION WITH A SHADOW ECONOMY

Table 1..4: Variables included in X_i

Variable	Description	Values
Worker characteristics		
Gender	Dummy variable equal to 1 for women	0-1
Age	Age of the worker	16-90
Age ²	Age squared	
Ed years	Number of education years	0-26
Degree	Highest degree achieved	1-5
		1 - no degree
		5 - postgraduate degree
Y work	Number of months worked in the last year	1-12
Experience	Number of months worked in the last job	0-720
First job	Dummy for the first job (1 if it is the first job)	0-1
Production unit (firm) characteristics		
Sector Man	Dummy for the manufacturing sector	0-1
Sector Fin	Dummy for financial intermediation	0-1
Sector ret	Dummy for the sales and retailers sector	0-1
Big city	Dummy for a firm in one of the two largest cities	0-1
Size	Categories for the number of workers	1-9
		1 - One worker
		9 - More than 101 workers
Production unit (Type of job) characteristics		
Lib	Dummy for a liberal occupation	0-1
Admin	Dummy for an administrative task	0-1
Seller	Dummy for sellers and related	0-1
Services	Dummy for a service task (bartender ..)	0-1
Worker-firm relationship		
Union	Dummy for labor union affiliation (1 if yes)	0-1
Agency	Dummy for agency hiring (1 if yes)	0-1
Seniority	Number of months of the worker in the firm	0-720

Table 1..5: Estimation results

Parameter	Point estimate	std. error	t-statistic	95% conf. interval	
γ_0^f	6.859	0.033	211.9	6.89	7.02
$\gamma_0^s - \gamma_0^f$	0.102	0.032	-3.2	-0.16	-0.04
γ_1^s	0.682	0.037	12.6	0.648	0.716
β -Gender	-0.077	0.005	-11.6	-0.06	-0.04
β -Age	0.025	0.001	13.1	0.01	0.02
β -Age ²	0.000	0.000	-8.8	0.00	0.00
β -Ed years	0.037	0.002	15.4	0.02	0.03
β -Degree	0.156	0.005	21.1	0.10	0.12
β -Sector Man	-0.098	0.006	-11.9	-0.08	-0.06
β -Sector Fin	0.156	0.015	6.9	0.08	0.14
β -Sector ret	-0.150	0.006	-16.9	-0.11	-0.09
β -Big city	0.010	0.007	1.0	-0.01	0.02
β -Size	0.032	0.001	18.7	0.02	0.02
β -Union	0.126	0.010	8.3	0.07	0.11
β -Agency	-0.144	0.005	-18.3	-0.11	-0.09
β -Seniority	0.001	0.000	17.9	0.00	0.00
β -Y work	0.029	0.001	18.4	0.02	0.02
β -First job	-0.053	0.008	-4.7	-0.05	-0.02
β -Experience	0.000	0.000	5.3	0.00	0.00
β -Lib	0.074	0.013	3.9	0.03	0.08
β -Admin	-0.272	0.009	-19.9	-0.20	-0.17
β -Seller	-0.186	0.014	-9.2	-0.15	-0.10
β -Services	-0.267	0.009	-19.3	-0.20	-0.16

Table 1..6: Pareto distribution estimates

Parameter	Point estimate	std. error	z-statistic	95% conf. interval	
Shape parameter	1.81	0.0018	953.34	1.806	1.833

2 Optimal Taxation with Permanent Employment Contracts

Abstract

New Dynamic Public Finance describes the optimal income tax in the economy without private insurance opportunities. I extend this framework by introducing permanent employment contracts which facilitate insurance provision within firms. The optimal tax system becomes remarkably simple, as the government outsources most of the insurance provision to employers and focuses mainly on redistribution. When the government wants to redistribute to the poor, a dual labor market could be optimal. Less productive workers are hired on a fixed-term basis and are partially insured by the government, while the more productive ones enjoy the full insurance provided by the permanent employment. Such arrangement can be preferred, as it minimizes the tax avoidance of top earners. I provide empirical evidence consistent with the theory and characterize the constrained efficient allocations for Italy.

2.1 Introduction

Lifetime incomes differ due to initial heterogeneity in earning potential of workers and luck experienced during the working life.¹ The standard welfare criteria call for the elimination of both types of inequality. New Dynamic Public Finance (NDPF) answers this call by designing a tax system that both redistributes income between initially different people and insures them against differential luck realizations.² This approach has been criticized for two reasons. First, it neglects private insurance possibilities. Second, the optimal tax system is far more complicated than any tax system observed in reality. In this paper I address these two problems of NDPF by introducing permanent employment contracts.

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¹ [Huggett, Ventura, and Yaron \(2011\)](#) estimate that out of the two, initial differences account for more than 60% of the inequality in lifetime earnings.

² [Golosov, Tsyvinski, and Werning \(2007\)](#) and [Kocherlakota \(2010\)](#) survey the NDPF literature.

The individual productivity of each worker evolves as a random process. Insuring a worker essentially means keeping his consumption constant through times of both high and low productivity. Insurance via income tax is difficult because the government does not observe individual productivity.³ I assume that firms have better information than government, yet face a different friction: neither they nor workers are able to commit to maintain the employment relationship. Permanent contracts with a high firing cost discourage employers from laying off their employees, thus allowing firms to act as insurers. The government optimally outsources most of the insurance to the better informed firms and, depending on social objectives, can focus on redistribution. As a result, the optimal tax system is simple: in the model calibrated to Italy any reasonable constrained efficient allocation can be implemented with a tax schedule that depends exclusively on current consumption expenditure.⁴ Such tax was proposed by various public finance economists in US.⁵ It contrasts with the standard implementation of NDPF which involves time-varying taxation of labor income and capital income that depends on the whole history of past earnings.

The insurance within firms comes at a price. Permanent contracts reduce the random variation of income over the life-cycle, but they also allow firms to shift workers' compensation across time in order to minimize the workers' tax burden. Such a behavior reduces the government's ability to redistribute. A redistributive government sometimes prefers to strip the least productive workers of the private insurance by equipping them with fixed-term contracts, since in this way they either receive higher transfers or face lower labor distortions. Hence, I provide a novel rationale for the coexistence of permanent and fixed-term contracts.

There is strong empirical evidence of income shifting within firm, both for insurance and tax avoidance reasons. [Guiso, Pistaferri, and Schivardi \(2005\)](#) document that Italian firms insure workers by reducing variability of their income. [Lagakos and Ordóñez \(2011\)](#) conduct a similar study for US and find that high-skilled workers obtain more insurance than low-skilled ones.⁶ [Kreiner, Leth-Petersen, and Skov \(2015\)](#) describe the shifting of salaries within firms in response to the announced decrease of the top income tax rate in Denmark. Individuals affected by the reform shifted on average 10% of their labor income, although the effect is concentrated in a relatively small group of taxpayers that shift most of their salaries. In a companion paper, [Kreiner, Leth-Petersen, and Skov \(2014\)](#) focus on top management. Managers are most likely to shift income by retiming bonus payments, but delaying the regular wage income is also evident.

In my model economy risk averse workers face risk due to stochastic idiosyncratic productivity

³Financial markets are also unlikely to observe individual productivity of every worker.

⁴By a reasonable allocation I mean the allocation that does not involve redistribution of income from the poor to the rich.

⁵The progressive consumption tax was advocated by [Hall and Rabushka \(1995\)](#) and [Bradford \(2000\)](#).

⁶Although there is no mandatory firing cost in US, [Bishow and Parsons \(2004\)](#) shows that between 1980 and 2000 on average 30% of employees in private establishments were covered by a voluntary severance pay provided by the employer. White collar workers are more likely to be covered, which can explain the gap in insurance between skill groups.

and can trade only a risk-free asset. Risk neutral firms observe workers' productivity and compete for them in the labor market. At first I consider the frictionless labor market, where workers and firms can credibly promise not to terminate the employment relationship. I show that the full commitment between workers and firms severely restricts the redistributive power of the state. For instance, when workers are risk neutral, only progressive tax schedules are incentive-compatible. If the tax was locally regressive, workers and firms would agree to randomize wages in order to reduce the average tax rate faced by the worker. Although the full commitment case is not realistic, it clearly shows that a reduction in contracting frictions between workers and firms exacerbates the tax avoidance and restricts redistribution. This observation will be useful in understanding the optimality of fixed-term contracts in the case without commitment on the labor market. The characterization of the full commitment case also sheds light on the generality of the optimal tax rate formulas expressed with sufficient statistics. [Chetty and Saez \(2010\)](#) show that the sufficient statistics formula for the linear income tax is valid also when workers have access to private insurance, as long as this insurance does not suffer from moral hazard. My results indicate that their finding cannot be generalized to the non-linear income tax. The optimal tax formulas derived by [Diamond \(1998\)](#) and [Saez \(2001\)](#) typically yield the U-shaped tax schedule, with marginal tax rates decreasing below the mode income. If the risk aversion of workers is sufficiently low, they could exploit the tax regressivity on low income levels by wage randomization.⁷

The main part of the paper is devoted to the frictional labor market, where neither workers nor firms are unable to commit to maintain the employment relationship in the future. Workers are free change employers. Firms can, at a specified cost, fire employees. I consider two different types of labor contract: permanent and fixed-term. Fixed-term contracts allow firms to dismiss workers in every period without any cost. Permanent contracts have high firing cost which discourages firms from laying off their workforce. When all workers have fixed-term contracts, the taxation problem is equivalent to NDPF. If a firm and a worker can terminate their relationship at no cost and start a new one with a clean slate, no private insurance is possible. Worker's income is equal to his output in each period and the labor market collapses to a sequence of spot labor markets. Optimally, the government steps in with taxation that both redistributes and insures. Since the government is constrained by available information, it has to set up a complicated, history dependent income tax system to screen evolving productivities of workers. [Goloso, Kocherlakota, and Tsyvinski \(2003\)](#) show that the optimal insurance provision with private information requires levying a tax on labor income and on savings, although agents are heterogeneous only in labor productivity. In the opposite case, when all workers are employed on a permanent basis, firms are not tempted to fire workers, but workers are unable to commit to stay in their firms. I show that this market

⁷Another striking example of the difference in optimal tax system with and without private insurance is the top tax rate. Consider the economy with a bounded productivity distribution. In the standard [Mirrlees \(1971\)](#) model the optimal top tax rate is non-positive. In contrast, in the model with full commitment on the labor market the optimal tax rate is positive when the government wants to redistribute towards the less productive workers.

imperfection can be remedied by backloading labor compensation. By shifting labor income to the future, employers effectively lock workers in the company. As workers no longer have incentives to quit, firms can offer them full consumption insurance.

I show that the workers that pay the highest taxes should always have permanent contracts and enjoy full consumption insurance. If they had fixed-term contracts instead, assigning them permanent contract would lead to a Pareto improvement for any tax system in place. The intuition is simple: with permanent contract, paying high taxes becomes more attractive. If this reform induced some other workers to change their behavior, they would end up contributing more resources to the governments budget. It could, nevertheless, be suboptimal to equip all workers with permanent contracts. When the government cares most about the initially least productive, these workers could optimally end up with fixed-term contracts and no private insurance. The reason behind this finding is as follows. Under permanent contracts firms can shift workers' income to the future. On the one hand, this allows firms to insure workers; on the other, firms have incentives to structure income payments in a way that minimizes their employees' tax burden. The currently productive workers would benefit from shifting income to the future and claiming transfers due to low current earnings. Since such income shifting is possible only under permanent contract, the government can prevent this by assigning fixed-term contracts at low levels of income. This argument provides a novel perspective on dual labor markets where the two types of contracts coexist, a prevalent labor market arrangement in Europe. There is the extensive literature documenting the negative impact of dual labor markets on the unemployment risk, the human capital accumulation and the volatility of business cycles.⁸ I complement this literature by showing how fixed term contracts influence individual responses to income taxation.

How to implement the optimal allocation with taxes? When all workers optimally have permanent contracts and full consumption insurance, they should face only the redistributive tax based on consumption expenditures.⁹ The usual base for redistributive tax, such as labor income or total income, exhibits time variation due to backloading of compensation. Since the consumption expenditures remain stable through a worker's lifetime, it allows the tax schedule to be time-invariant. I show that the tax is governed by a well-understood [Saez \(2001\)](#) formula from the static [Mirrlees \(1971\)](#) model.¹⁰ The tax schedule depends on the average lifetime elasticity of labor supply and only the initial distribution of types. Intuitively, if all people entered the labor market with an identical initial productivity and the same distribution of future shocks, any inequality in income would be a matter of insurance, not redistribution. Hence, it would be dealt with by firms. When

⁸See references in the related literature section. For information on dual labor markets in Europe, see [Eichhorst \(2014\)](#).

⁹The tax system described in this paragraph implements the optimum, unless the planner wants to redistribute income from the bottom to the top. In such unusual cases this implementation can yield a suboptimal outcome.

¹⁰Recall that the optimal tax system with full commitment on the labor market is not consistent with the [Saez \(2001\)](#) formula due to the threat of wage randomization. Introduction of the limited commitment on the side of workers is enough to prevent the wage randomization and recover the sufficient statistics formula.

tax payments increase progressively with consumption expenditures, the tax schedule can depend only on current consumption expenditures - no history dependence is required. Furthermore, there is no need to tax the savings of permanent workers. When the dual labor market is optimal, fixed-term workers are covered by an extensive public insurance program. As in NDPF, it involves a tax on savings that can be interpreted as means tested income support.

This paper focuses on the relation between the type of contract and the volatility of workers' income. I show that this effect is present in the data by analyzing the administrative records of employment histories from Italy. The residual income variance of a median worker is higher by 78% under fixed-term rather than permanent contract. This estimate is conditional on continuous employment at one firm, so it is not affected by income changes due to losing or switching jobs. I am the first to document the impact of fixed-term contracts on income volatility, conditional on staying employed. A proper causal analysis of the link between firing costs and income risk is an interesting topic for future research.

I calibrate a simple life-cycle model to Italy. All constrained efficient allocations involve assigning permanent contracts to all workers. As a result, all allocations at the Pareto frontier which do not involve redistribution from the bottom to the top can be implemented with a simple consumption expenditure tax. The welfare gains are substantial: when the planner is utilitarian, permanent contracts increase welfare gains from optimal taxation by 50%.¹¹ Then I investigate under which parameter values the dual labor market would be optimal. If the productivity of the initially least productive type was lower by at least 4%, the Rawlsian planner would assign fixed-term contract to these workers.

Related literature. This paper contributes to the literature on optimal taxation with private insurance markets. [Golosov and Tsyvinski \(2007\)](#) study this question under the assumption that the government and firms face the same friction: asymmetric information. I assume that frictions faced by firms and those faced by the government are different: the government lacks information, while firms and workers lack commitment. [Stantcheva \(2014\)](#) considers an environment in which firms face both limited information and limited commitment, but her model is static and hence concerned only with redistribution. [Chetty and Saez \(2010\)](#) model private insurance in the reduced form. Instead, my paper provides microfoundations of insurance on the labor market, which reveals the crucial role of the firing cost. [Attanasio and Rios-Rull \(2000\)](#) and [Krueger and Perri \(2011\)](#) study how the public insurance crowds out the private one. Although their private insurance is also constrained by the limited commitment friction, agents' endowments are random and exogenous. In my framework productivity is random, but income is endogenous. Shifting income across time

¹¹Suppose that utilitarian welfare of laissez-faire allocation is 100 in consumption equivalent terms. NDPF achieves 102.8, while optimal taxation with permanent contracts 104.3 (see Table 2.1). The permanent contracts regime improves NDPF relative to the laissez-faire by more than 50%.

turns out to be the key margin of response to taxes. In a different framework [Ábrahám, Koehne, and Pavoni \(2016\)](#) show that hidden asset trades reduce the optimal progressivity of labor income tax. I find that income shifting, which is related to asset trades, reduces the redistribution possible via the income tax.

Another strand of the literature focuses on simple tax implementations. [Albanesi and Sleet \(2006\)](#) show that with iid productivity shocks the constrained efficient allocations in NDPF can be implemented with potentially time-varying tax that depends jointly on current wealth and current labor income. [Farhi and Werning \(2013\)](#) and [Weinzierl \(2011\)](#) argue that age dependent taxation captures most of the welfare gains from the optimal non-linear taxes. [Findeisen and Sachs \(2015\)](#) optimize with respect to the history-independent, non-linear labor income tax and linear capital income tax rate. [Conesa, Kitao, and Krueger \(2009\)](#) is an example of a Ramsey approach, which restricts the tax function to some exogenously chosen class. My paper shows that the inclusion of private insurance leads to the fully optimal tax systems that are as simple as the tax functions assumed in the Ramsey approach.

Dual labor markets and fixed-term contracts are studied extensively. It was shown that temporary contracts are associated with higher unemployment risk ([García-Pérez, Marinescu, and Castello \(2014\)](#)) and lower on the job training ([Cabrales, Dolado, and Mora \(2014\)](#)) than permanent contracts. Furthermore, dual labor markets amplify macroeconomic fluctuations, as employers are less likely to hoard labor ([Bentolila, Cahuc, Dolado, and Le Barbanchon \(2012\)](#); [Kosior, Rubaszek, and Wierus \(2015\)](#)). I contribute to this literature by documenting that, conditional on continuous employment at one company, fixed-term workers have significantly more volatile income than permanent employees.

The labor market in my model is frictional, as both parties can terminate the contract at any time. There is a long tradition of modeling labor market without commitment, dating back at least to [Harris and Holmstrom \(1982\)](#) and [Thomas and Worrall \(1988\)](#). [Thomas and Worrall \(2007\)](#) provide a recent review of the limited commitment models of labor market. This friction plays a key role also in other insurance markets: life insurance ([Hendel and Lizzeri \(2003\)](#)) and health exchanges ([Handel, Hendel, and Whinston \(2013\)](#)).

Structure of the paper. The next section introduces the environment and sets up the taxation problem. The optimum with full commitment on the labor market is characterized in Section 2.3. Section 2.4 characterizes the constrained efficient allocation with limited commitment. Implementation with a tax system is discussed in Section 2.5. In Section 2.6 I validate the predictions of the model with Italian data. The model is calibrated to Italy in Section 2.7. The last section concludes. All proofs are available in the Appendix.

2.2 Framework

In this section I describe the structure of the labor market, define the equilibrium and set up the optimal taxation problem.

2.2.1 Workers and firms

There is a continuum of workers that live for $\bar{t} \in \mathbb{N}_+$ periods. In each period they draw a productivity, which I describe in detail below. A worker with productivity θ and labor supply n produces output θn . Workers sell their labor to firms in exchange for a labor income y . Workers have access to the risk-free asset, in which they can save and borrow up to the limit $b \geq 0$ at the gross interest rate R . I denote a worker's choice of assets by a and assume that workers have no wealth initially. A worker's contemporaneous utility depends on consumption and labor supply according to a twice differentiable function $U(c, n) = u(c) - v(n)$, where u is increasing and strictly concave, while v is increasing and strictly convex. A worker's lifetime utility is a discounted expected stream of contemporaneous utilities, where $\beta = R^{-1}$ is a discount factor.

There is a continuum of identical firms. Firms maximize expected profits by hiring workers, compensating them with labor income and collecting output. Firms observe each worker's productivity and labor supply. I assume no entry cost for firms.

The labor market operates in the following way. Workers enter the market after their initial productivity is drawn. Firms make them offers which specify the labor supply and the labor income at each productivity history. I assume no search friction - all workers see all the offers immediately - which leads to a Bertrand competition between firms for workers. Once the contract is signed, the terms of the contract cannot be changed.¹² However, the contract can be terminated at will by both parties. At any point in time workers are free to leave their current employer and start a new job elsewhere. Workers face no mobility cost. On the other hand, firms can fire their employees in any period subject to the specified firing cost.¹³ I restrict the firing cost, denoted by f , to belong to the set $\{0, \bar{f}\}$, where \bar{f} is set sufficiently high such that no firm would ever be tempted to fire the worker. I will use the firing cost to distinguish between the *permanent workers* (those for which $f = \bar{f}$) and the *fixed-term workers* ($f = 0$).

¹²It is an important assumption. If firms were unable to commit to the terms of the contract, the equilibrium would involve no private insurance regardless of the firing cost.

¹³One can think about the firing cost as a severance payment to the fired worker. In the setting I consider such interpretation plays no role, as no firing is going to happen in equilibrium.

2.2.2 Productivity histories

In each period t (where $1 \leq t \leq \bar{t}$) a worker draws productivity from a finite set $\Theta_t \subset \mathbb{R}_+$. A history is a tuple of consecutive productivity draws starting at the initial period. The length of history h - the number of productivity draws it contains - is denoted by $|h|$. The history h belongs to the set $\Theta^{|h|} = \prod_{t=1}^{|h|} \Theta_t$ and the set of all histories is $\Theta \equiv \cup_{t=1}^{\bar{t}} \Theta^t$. Since all histories start in period 1, the length of the history is also the current time period. The i -th element of history h is h_i and the tuple of its first i elements is $h^i = (h_1, \dots, h_i)$. In order to simplify notation, I denote the last productivity at the history h as $\theta(h) \equiv h_{|h|}$ and the history directly preceding the history h as $h^{-1} \equiv h^{|h|-1}$. For clarity, consider the following example:

$$h = (\theta_a, \theta_b, \theta_c) \in \Theta^3, |h| = 3, h^{-1} = (\theta_a, \theta_b), \theta(h) = \theta_c.$$

The probability of drawing some history h of length t is equal $\mu(h)$ which is non-negative and sums up to 1 for all histories of this length: $\forall_t \sum_{s \in \Theta^t} \mu(s) = 1$. In practice, I will work mostly on the collections of histories that happen with positive probability, denoted by $\mathcal{H} \equiv \{h \in \Theta : \mu(h) > 0\}$. \mathcal{H}_t is the set of histories of length t that happen with positive probability. By $\mathcal{X}(h)$, where \mathcal{X} is a set of histories and $h \in \mathcal{H}$, I denote the subset of elements of \mathcal{X} that contain h : $\mathcal{X}(h) = \{s \in \mathcal{X} : s^{|h|} = h\}$. Specifically, $\mathcal{H}_t(h)$ is the set of possible histories of length t that contain sub-history h . The probability of drawing history $s \in \mathcal{H}(h)$ conditional on history h , where $\mu(h) > 0$, is denoted by $\mu(s | h)$. I assume that each initial type faces the productivity risk: $\forall_{\theta \in \Theta_1} \forall_{h \in \mathcal{H}_{\bar{t}}(\theta)} \mu(h | \theta) < 1$.

Definition 2.1. The allocation (c, y, n) specifies consumption $c : \mathcal{H} \rightarrow \mathbb{R}_+$, labor income $y : \mathcal{H} \rightarrow \mathbb{R}$ and labor supply $n : \mathcal{H} \rightarrow \mathbb{R}_+$ at each history.

Now we can specify the payoffs of agents. The expected utility of a worker at the history $h \in \mathcal{H}$, given the allocation (c, y, n) is

$$\mathbb{E}U_h(c, n) \equiv \sum_{s \in \mathcal{H}(h)} \mu(s | h) \beta^{|s|-|h|} U(c(s), n(s)). \quad (2.1)$$

Profits from hiring a worker at the history h given the allocation (c, y, n) are

$$\mathbb{E}\pi_h(y, n) \equiv \sum_{s \in \mathcal{H}(h)} \mu(s | h) R^{|h|-|s|} (\theta(s) n(s) - y(s)). \quad (2.2)$$

I denote the expected payoffs of workers from the *ex ante* perspective by dropping the superscript: $\mathbb{E}U(c, n) \equiv \sum_{\theta \in \Theta_1} \mu(\theta) \mathbb{E}U_\theta(c, n)$, and analogously for firms.

2.2.3 The social planner

I assume that the social planner observes consumption c , labor income y and the firing cost f , but does not observe the productivity θ , hours worked n and individual output θn . The distinction between the observable labor income y and the unobservable output θn is realistic and crucial for modeling the insurance and the tax avoidance within firm. If the worker was paid his output in every period, there would be no insurance on the labor market. If the planner observed not only labor income, but also output, firms would not be able to use income shifting to reduce the tax burden of workers. The social planner sets up a mechanism which governs the allocation of resources in the economy. By the revelation principle, without the loss of generality we can focus our attention on direct mechanisms.

Definition 2.2. A *direct mechanism* $(\mathcal{H}, (c, y, f))$ consists of the message space \mathcal{H} and the outcome functions (c, y, f) , each going from \mathcal{H} to a relevant subset of \mathbb{R} .

The planner in each period collects type reports of workers and assigns them consumption levels, labor incomes and firing costs. The agents' reports and the unobserved labor supply are determined in the equilibrium corresponding to the chosen mechanism. From now on I fix the message space at \mathcal{H} and identify a given mechanism with its outcome functions (c, y, f) .

Let's formalize the possible reporting behavior of agents. The pure reporting strategy r is a function from the set of possible histories to the message space: $r : \mathcal{H} \rightarrow \mathcal{H}$. I impose the consistency condition: $\forall_{s, h \in \mathcal{H}} s \in \mathcal{H}(h) \implies r(s) \in \mathcal{H}(r(h))$. It means that consecutive history reports cannot be at odds with which histories are in fact possible. Let's denote the set of consistent pure reporting strategies by \mathcal{R} . The truthful reporting strategy r^* is an identity: $r^*(h) = h$ for all $h \in \mathcal{H}$. I allow for mixed reporting strategies $\sigma \in \Delta_{\mathcal{R}}$, where σ is a probability distribution over the pure reporting strategies. The distribution assigning all the probability mass to the truthful reporting strategy r^* is denoted by σ^* .

The expected utility of a worker at the history h , given outcome functions (c, y) , a pure reporting strategy r and a corresponding labor allocation n_r is $\mathbb{E}U_h(c \circ r, n_r)$, where $c \circ r$ is a composite function of reporting strategy and consumption function: $(c \circ r)(h) = c(r(h))$. Similarly, the firm's profits are $\mathbb{E}\pi_h(y \circ r, n_r)$. Therefore, the reporting strategy directly affects the outcomes that are assigned by the mechanism. The payoffs of a worker and a firm from the mixed reporting strategy $\sigma \in \Delta_{\mathcal{R}}$ and the corresponding labor allocation $n_\sigma = \{n_r : \mathcal{H} \rightarrow \mathbb{R}_+\}_{r \in \mathcal{R}}$ at history h are $\sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}U_h(c \circ r, n_r)$ and $\sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}\pi_h(y \circ r, n_r)$. Note that in the case of the mixed reporting strategy, the labor supply allocation is allowed to vary with the selected pure reporting strategy.

2.2.4 Equilibrium

Since all firms are identical, there is no gain from workers changing employers. Hence, without the loss of generality, I focus on the equilibria without separations. The following lemma describes the conditions such equilibria have to satisfy.

Lemma 2.1. *(σ, n_σ) is such that neither a worker nor a firm has incentives to terminate the employment relationship if and only if*

$$\forall_{r \in \mathcal{R}} \text{ s.t. } \sigma(r) > 0 \quad \forall_{h \in \mathcal{H}} -f(r(h)) \leq \mathbb{E}\pi_h(y \circ r, n_r) \leq 0. \quad (2.3)$$

The worker has incentives to leave if the employer makes positive profits on him. If that happens, a competing firm could offer the worker a better deal, while still being profitable. On the other hand, the firm has incentives to fire the worker if the expected losses are greater than the firing cost. The limited commitment constraints (2.3) prevent both deviations.

Definition 2.3. (σ, n_σ) is the equilibrium of mechanism (c, y, f) if (i) (σ, n_σ) satisfies (2.3) and (ii) there is no other $(\sigma', n'_{\sigma'})$ which satisfies (2.3) and additionally $\sum_{r \in \mathcal{R}} \sigma'(r) \mathbb{E}U(c' \circ r, n'_r) > \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}U(c \circ r, n_r)$ and $\sum_{r \in \mathcal{R}} \sigma'(r) \mathbb{E}\pi(c' \circ r, n'_r) \geq \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}\pi(c \circ r, n_r)$.

The equilibrium reporting strategy and the labor supply allocation are determined by the payoff maximizing behavior of workers and firms, given the competition and the limited commitment on the labor market. Specifically, there can be no other $(\sigma', n'_{\sigma'})$ which is consistent with the limited commitment constraints, yields not lower profits to firms and strictly greater utility to workers. The following lemma describes the set of equilibria of a mechanism.

Lemma 2.2. *The set of equilibria of mechanism (c, y, f) is*

$$\mathcal{E}(c, y, f) \equiv \arg \max_{\substack{\sigma \in \Delta_{\mathcal{R}} \\ \{n_r : \mathcal{H} \rightarrow \mathbb{R}_+\}_{r \in \mathcal{R}}}} \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}U(c \circ r, n_r),$$

where maximization is subject to the limited commitment constraints (2.3) and the zero profit condition

$$\forall_{\theta \in \Theta_1} \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}\pi_\theta(y \circ r, n_r) = 0. \quad (2.4)$$

Since workers observe all offers, the competition between firms for workers drives profits to zero. Notice that the zero profit condition means that firms cannot redistribute. Any transfer of resources between initial types would mean that the firm is making profit on one type and losses on another. It cannot happen in equilibrium, as the profitable type would be captured by the competing firm.

Definition 2.4. The mechanism (c, y, f) *implements* allocation (c, y, n) if $(\sigma^*, n) \in \mathcal{E}(c, y, f)$, i.e. if labor supply allocation n and the truthful reporting strategy constitute the equilibrium of the mechanism (c, y, f) .

Note that the above notion of implementation does not require (σ^*, n) to be the unique equilibrium of the mechanism. The optimal mechanisms generally have multiple equilibria, some of which involve untruthful reporting. I implicitly assume that if there exists a truthful equilibrium, the agents will choose it.¹⁴

2.2.5 The planner's problem

The planner chooses the mechanism in order to maximize the social welfare function

$$\max_{\substack{c : \mathcal{H} \rightarrow \mathbb{R}_+ \\ y : \mathcal{H} \rightarrow \mathbb{R} \\ f : \mathcal{H} \rightarrow \{0, \bar{f}\}}} \sum_{\theta \in \Theta_1} \lambda(\theta) \mu(\theta) \mathbb{E}U(c, n), \quad (2.5)$$

where λ is the non-negative Pareto weight with the expected value of 1: $\sum_{\theta \in \Theta_1} \lambda(\theta) \mu(\theta) = 1$. The optimization is subject to the resource constraint

$$\sum_{h \in \mathcal{H}} R^{1-|h|} \mu(h) (y(h) - c(h)) \geq 0 \quad (2.6)$$

and the equilibrium constraint

$$(\sigma^*, n) \in \mathcal{E}(c, y, f). \quad (2.7)$$

The equilibrium constraint means that the chosen mechanism (c, y, f) implements the allocation (c, y, n) . It incorporates the usual incentive compatibility constraints that prevent type misreporting. Note that the untruthful equilibria in $\mathcal{E}(c, y, f)$ correspond to the binding incentive constraints.

2.3 Frictionless labor market

In this section I solve the government problem under assumption of private sector operating without frictions: both workers and firms can commit to maintain the employment relationship. However, I do not allow firms and workers to contract before the initial productivity draw. If contracting behind the veil of ignorance was allowed, as in [Goloso and Tsyvinski \(2007\)](#), firms could redistribute between initial types. Here redistribution within firm is prevented, as any labor contract involving

¹⁴It is a usual assumption in the literature. Without it, the planner's problem could have no solution. Note that payoffs of workers and firms are identical for any contract in $\mathcal{E}(c, y, f)$.

cross-subsidization allows competitors to profitably steal the worker that is paid less than his product.

Corollary 3. *Under full commitment, the set of equilibrium contracts is*

$$\mathcal{E}^{FC}(c, y) \equiv \arg \max_{\substack{\sigma \in \Delta_{\mathcal{R}} \\ \{n_r : \mathcal{H} \rightarrow \mathbb{R}_+\}_{r \in \mathcal{R}}}} \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}U(c \circ r, n_r), \text{ s.t. } \forall_{\theta \in \Theta_1} \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}\pi_{\theta}(y \circ r, n_r) = 0.$$

Since firms and workers can credibly commit to maintain the employment relationship, we can drop the limited commitment constraints. This means that the firing cost does not influence the equilibrium. The initial zero profit condition becomes the sole constraint in determination of the equilibrium contract.

In equilibrium the firm chooses the labor supply policy that minimizes the disutility cost of working conditional on satisfying the zero profit condition. Suppose that at the equilibrium reporting strategy the expected lifetime income of initial type h_1 is $Y(h_1)$. The necessary and sufficient condition for the optimal labor supply of each initial type is to equalize the marginal cost of output across all histories and reporting strategies

$$\forall_{r \in \mathcal{R}} \forall_{h \in \mathcal{H}} \frac{v'(n_r(h))}{\theta(h)} \equiv \phi_{h_1}(Y(h_1)). \quad (2.8)$$

Under full commitment the equilibrium allocation of labor supply produces the expected lifetime income Y at the minimal disutility cost. The output and the labor income agree in expectations over the lifetime of a worker, which is captured by the zero profit condition, but do not have to coincide at every history. It means that the firm can shift worker's income across time and productivity histories. The output produced by worker of initial type θ at some history $h \in \mathcal{H}(\theta)$ can be paid to him at any other history $h' \in \mathcal{H}(\theta)$, as long as the zero profit condition (2.4) holds. Such an unrestricted income shifting is possible only because of the full commitment of firms and workers.

Let's denote by $n_Y^{FC} : \mathcal{H} \rightarrow \mathbb{R}_+$ the labor supply allocation which satisfies the optimality condition (2.8) and generates the expected lifetime income Y . We can use it to construct the indirect utility function that captures the expected utility of some initial type θ from lifetime consumption C and lifetime labor income Y :

$$V_{\theta}(C, Y) \equiv \bar{\beta} u(C/\bar{\beta}) - \sum_{h \in \mathcal{H}(\theta)} \beta^{|h|-1} \mu(h | \theta) v(n_Y^{FC}(h))$$

where $\bar{\beta} \equiv \sum_{t=1}^{\bar{t}} \beta^{t-1}$. Notice that $V_{\theta}(C, Y)$ implicitly assumes that the worker enjoys full consumption insurance, while the labor supply is chosen in order to minimize the disutility cost of

producing the lifetime income Y . By the theorem below, we can use this indirect utility function to simplify the taxation problem under full commitment.

Theorem 2.1. *Under full commitment on the labor market, all workers enjoy full consumption insurance and the planner's problem can be expressed as*

$$\max_{(C(\theta), Y(\theta))_{\theta \in \Theta_1}} \sum_{\theta \in \Theta_1} \lambda(\theta) \mu(\theta) V_{\theta}(C(\theta), Y(\theta))$$

subject to the resource constraint

$$\sum_{\theta \in \Theta_1} \mu(\theta) (Y(\theta) - C(\theta)) \geq 0$$

and the incentive-compatibility constraints

$$\forall \theta, \theta' \in \Theta_1 \quad \bar{\beta} u(C(\theta)/\bar{\beta}) - Y(\theta) \phi_{\theta}(Y(\theta)) \geq \bar{\beta} u(C(\theta')/\bar{\beta}) - Y(\theta') \phi_{\theta}(Y(\theta)). \quad (2.9)$$

Under full commitment on the labor market firms are ideal insurers of workers. Firms are driven by competition to provide workers with the maximal utility attainable without making losses. Moreover, they are better informed than the planner. The optimal mechanism makes use of firms to provide full consumption insurance to workers.

By Theorem 2.1, the planner chooses only the lifetime consumption and lifetime labor income of each initial type. Since all agents enjoy constant consumption, a lifetime consumption fully determines the consumption at each history. Furthermore, because of full commitment, the allocation of labor in equilibrium depends only on the expected lifetime income and not on labor income on any particular history. No matter how the labor income is structured by the planner, the firm will always allocate labor to histories in a way that minimizes the total disutility cost of lifetime production.

The incentive compatibility constraints (2.9) are not standard. The reason is that in the optimum the worker is never tempted by reporting some other type with certainty. In the proof of Theorem 2.1 I show that if that was the case, then the worker would be strictly better off with mixing between this reporting strategy and truth-telling. The mixed reporting strategy is better, because under full commitment the firm can equalize the marginal cost of production across pure reporting strategies over which the worker randomizes. Since the disutility from labor is strictly convex, the worker strictly gains from this labor smoothing across reporting strategies. Consequently, only the incentive constraints corresponding to the mixed strategies can bind in the optimum. Condition (2.9) means that the gain from a marginal increase in probability of reporting θ' when the true type is θ is non-positive. It is a necessary and sufficient condition for truth-telling when workers

can use mixed reporting strategies.

Since the incentive constraints with respect to all pure reporting strategies need to be slack, the truth-telling places a tight constraint on implementable allocations. To see this, define a lifetime tax $T(Y(\theta)) \equiv Y(\theta) - C(\theta)$ and its marginal rate $T'(Y(\theta)) \equiv 1 - \frac{\phi_\theta(Y(\theta))}{u'(C(\theta)/\beta)}$.

Proposition 2.1. *Under full commitment on the labor market, the mechanism $(C(\theta), Y(\theta))_{\theta \in \Theta_1}$ implements truth-telling only if $(1 - T'(Y))u' \left(\frac{Y - T(Y)}{\beta} \right)$ is non-increasing in Y .*

Full commitment on the labor market restricts the regressivity of the tax schedule. For instance, when workers are risk neutral, only progressive (i.e. convex) tax schedules are implementable. Suppose on the contrary that the tax schedule is strictly regressive on some income interval $[Y, Y']$. It means that the marginal tax rate at Y is greater than the average tax rate at this interval:

$$T'(Y) > \frac{T(Y') - T(Y)}{Y' - Y}. \quad (2.10)$$

By substituting the definitions of the tax and the marginal tax rate one can show that the incentive compatibility constraint (2.9) is violated. Suppose that the worker with income Y marginally increases output. How should a firm compensate the worker? The additional income can be paid with certainty and taxed at the marginal tax rate. Alternatively, the firm can compensate the worker with additional income $Y' - Y$ which is paid with probability low enough such that the firm makes no losses. This additional income is taxed at the average tax rate at the income interval $[Y, Y']$. Whenever the average rate is lower than the marginal rate, i.e. whenever the tax schedule is strictly regressive, the risk-neutral worker strictly prefers random compensation. For risk averse workers such income randomization is naturally less attractive, hence some degree of regressivity is still possible.

Chetty and Saez (2010) show that the sufficient statistics formula for the optimal linear income tax is valid also in the presence of private insurance, as long as private insurers do not suffer from moral hazard. In my framework the private insurance is free of moral hazard, since firms observe workers' types. Hence, by Proposition 2.1 the result of Chetty and Saez (2010) does not generalize to a non-linear income tax. The sufficient statistic formula for the optimal non-linear income tax may prescribe a locally regressive tax. In fact, the optimal taxation literature typically recommends the U-shaped tax schedules, with tax rates decreasing below the mode income (see Diamond (1998); Saez (2001)).¹⁵ If the labor market operated under full commitment and the risk aversion was sufficiently low, such tax would induce the tax avoidance via income randomization at low levels of income. As a result, the tax revenue would be lower than predicted.

¹⁵Such recommendations are drawn from the static Mirrlees (1971) model. It is a special case of my framework, when workers live for only one period ($\bar{t} = 1$) and the commitment on the labor market is limited.

Another way to see that the optimal tax rate under full commitment is qualitatively different than the optimal tax rate without private insurance is to consider the top tax rate. In the static [Mirrlees \(1971\)](#) model this rate is always non-positive. In the model with the full commitment on the labor market this rate will be positive when the incentive constraint of the top type binds. That is the case because decreasing labor supply of the top type reduces his utility from the marginal deviation to a mixed reporting strategy.

Proposition 2.2. *Suppose that workers live for one period ($\bar{t} = 1$) and the planner is utilitarian. In the optimum, the labor supply of the top type is distorted downwards.*

2.4 Frictional labor market

In this section I characterize the optimal allocation when the labor market is frictional: workers can leave firms and firms can fire workers, subject to the firing cost. From the previous section we know that under full commitment on the labor market the set of implementable allocations is severely constrained by the possibility of using mixed reporting strategies. Without commitment on the side of workers mixed strategies are much less powerful and we can focus exclusively on pure reporting strategies.

Lemma 2.3. *Under limited commitment on the labor market, the payoff from any mixed reporting strategy is dominated by the payoff from some pure reporting strategy.*

With full commitment workers could smooth labor across the different pure reporting strategies over which they were mixing. At some pure strategies the firm made positive profits, at others - suffered losses. Without commitment on the workers' side such arrangement is not sustainable, as workers have incentives to leave the firm if it makes strictly positive profits. Hence, the limited commitment of workers prevents firms from reducing a tax burden via the wage randomization.

Without commitment on the labor market the type of labor contract matters, since the high firing cost prevents firms from dismissing their workers. The following two subsections describe the optimal allocation when the planner is restricted to use only fixed-term or only permanent contracts. Finally I describe the optimal choice of the contract type.

2.4.1 Only fixed-term contracts

Suppose that the planner assigns fixed-term contracts to workers at each history: $\forall_{h \in \mathcal{H}} f(h) = 0$.

Lemma 2.4. *Under fixed-term contracts, in any equilibrium (r, n) at any history $h \in \mathcal{H}$ the worker's labor income is equal to the worker's output: $y(r(h)) = \theta(h)n(h)$.*

The zero firing cost under fixed-term contracts means that neither firm nor worker can commit to maintain the employment. This lack of commitment implies that neither of the parties can owe any resources to another, as such a loan would never be repaid. As a result, the labor market becomes a sequence of spot labor markets: a worker at each history is paid exactly his current output.

Corollary 4. *Under fixed-term contracts, the planner's problem is a New Dynamic Public Finance taxation problem.*

Lemma 2.4 tells us that the reporting strategy uniquely determines the equilibrium labor supply policy, since output equals labor supply in each period. Hence, we can reformulate the equilibrium constraint (2.7) as

$$\forall_{r \in \mathcal{R}} \mathbb{E}U(c, \bar{n}(r^*)) \geq \mathbb{E}U(c \circ r, \bar{n}(r)), \text{ where } \forall_{h \in \mathcal{H}} \bar{n}(r(h)) = \frac{y(r(h))}{\theta(h)}. \quad (2.11)$$

This is exactly the incentive-compatibility constraint considered by NDPF. Since the firms do not insure their workers, the government steps in with the tax system which both redistributes and insures. As the planner is limited by information, the consumption insurance is only partial. [Goloso, Kocherlakota, and Tsyvinski \(2003\)](#) show that workers' consumption evolves according to the inverse Euler equation, which implies a downward distortion of savings. More recently [Farhi and Werning \(2013\)](#) and [Goloso, Troshkin, and Tsyvinski \(2016\)](#) provide the detailed characterization of the optimal labor wedges.

2.4.2 Only permanent contracts

Suppose that the planner uses only permanent contracts: $\forall_{h \in \mathcal{H}} f(h) = \bar{f}$. The firing cost in this case is assumed to be so high that no firm is ever tempted to fire a worker. Since workers are still free to leave the firm, the labor market operates under one-sided lack of commitment. The equilibria in the similar settings were characterized by [Harris and Holmstrom \(1982\)](#) and [Krueger and Uhlig \(2006\)](#).¹⁶ The firm overcomes a worker's commitment problem by backloading labor compensation, i.e. shifting it to the future. As the reward for work comes in the later periods, workers have less incentives to leave the employment relationship early.

Theorem 2.2. *Take any allocation of consumption and labor supply that can be implemented under the full commitment on the labor market. The planner can implement it under permanent contracts.*

¹⁶In [Harris and Holmstrom \(1982\)](#) a firm and a worker learn symmetrically about the worker productivity. They receive noisy signals and the contract is based on the posterior mean of productivity. As the posterior mean is a random variable, this model is equivalent to the framework considered in this paper, where the productivity is observable, but stochastic. [Krueger and Uhlig \(2006\)](#) analyze risk-sharing contracts between risk neutral intermediaries and risk averse agents with risky endowments.

This is one of the main results of this paper. Although the labor market is frictional, as workers cannot credibly promise to stay with their employers, the planner still can provide workers with full consumption insurance.¹⁷ The reasoning is simple due to a direct mechanism approach. The utility of workers depends on their allocation of consumption and not on the allocation of labor income. The limited commitment constraints, on the contrary, depend on labor income but not on consumption. This means that the limited commitment constraints can be relaxed by backloading labor income without affecting the consumption allocation.

We can understand this result in the following way. The firm offers a labor income that is increasing in tenure and varies only with the initial productivity realization. This contract will satisfy the limited commitment constraints, as the labor income is backloaded. The initial compensation can be adjusted such that the firm makes no losses in expectations. Given that the compensation is deterministic, workers can smooth their consumption perfectly by borrowing against future labor income. If the required borrowing is not available due to the borrowing limit, the consumption can be smoothed with age or tenure dependent taxation.

Corollary 5. *For any Pareto weights $\{\lambda(\theta)\}_{\theta \in \Theta_1}$, the optimum without commitment on the labor market yields weakly higher social welfare than the optimum with full commitment. The relation is strict if the optimum with full commitment features binding incentive constraints.*

The first statement is a simple implication of Theorem 2.2. The second statement comes from the fact that the full commitment optimum the binding incentive constraints corresponding to the mixed strategies. However, Lemma 2.3 shows that without commitment on the labor market these constraints become slack. Hence, without commitment between firms and workers the redistributive planner is less constrained by tax avoidance and can achieve higher social welfare.

Although the planner can implement full consumption insurance, it will not always be desirable to do so, even when all workers have permanent contracts. In the next subsection I discuss cases in which the planner optimally assigns different types of contracts to different workers, effectively stripping some of them of insurance. Under some circumstances such a dual labor market allocation can be implemented even if all types are nominally assigned permanent contracts.¹⁸ For an example of such a situation, see Lemma 2.9 in the Appendix.

¹⁷Harris and Holmstrom (1982) showed that workers can receive full consumption insurance when sufficient borrowing is available (see their footnote 5). My result is more general, as it holds irrespectively of the workers' borrowing limit.

¹⁸A dual labor market allocation is preferable, because fixed-term contract prevents income shifting of the deviating type. However, in some cases the limited commitment constraints of worker are enough to prevent the income shifting. That is the case when the deviating worker wants to work less in the first period and more in the second. For details, see Lemma 2.9.

2.4.3 Who should have permanent contract?

In the following two subsections I investigate which workers should have permanent contracts, and which fixed-term contracts. Theorem 2.3 states that workers that pay the highest taxes should have permanent contracts.

Definition 2.5. *An initial top taxpayer is a type that belong to*

$$\arg \max_{\theta \in \Theta_1} \sum_{s \in \mathcal{H}(\theta)} R^{1-|s|} \mu(s | \theta) (y(s) - c(s)).$$

Theorem 2.3. *Initial top taxpayers optimally have permanent contracts and full consumption insurance.*

Assigning permanent contracts allows the planner to provide more insurance and save resources, but it also increases the incentives of other workers to misreport. However, there are some types that can be mimicked without a loss for the planner: initial top taxpayers. If any other initial worker decides to report that he is a top taxpayer, he will end up contributing more resources to the planner's budget. Hence, simply assigning permanent contracts to top taxpayers is a Pareto improving reform. Note that top taxpayers need not be top earners. If the planner cares only about the most productive types, the least productive workers are taxed the most and they should receive permanent contracts. Theorem 2.3 leads us to a strong conclusion: it is never optimal to assign fixed-term contracts to all workers. The planner can always Pareto improve upon the NDPF allocation by introducing permanent employment contracts.

Corollary 6. *If the planner does not want to redistribute between initial types, all workers are optimally assigned permanent contracts and full consumption insurance.*

In the particular case of no redistribution all initial types are top taxpayers. If a planner cares only about insurance, it is optimal to use only permanent contracts.

2.4.4 Who should have fixed-term contract?

Take some allocation (c, y, n) with corresponding contract assignment f where the initial type $\underline{\theta}$ has permanent contract. Consider an alternative contract assignment f' where the worker at the history $\underline{\theta}$ (and all histories that follow) receives a fixed-term contract and the contract types of other workers are unchanged. Denote the best allocation of consumption and labor, conditional on contract assignment f' , by (c', n') . Let's write the social welfare function as $W(c, n) \equiv \sum_{\theta \in \Theta_1} \lambda(\theta) \mu(\theta) \mathbb{E} U_{\theta}(c, n)$. We can decompose the welfare impact of switching contract type into three components, capturing

the change in efficiency, redistribution and insurance:

$$W(c', n') - W(c, n) = \underbrace{W(c', n') - W(c_2, n)}_{\Delta^{efficiency}} + \underbrace{W(c_2, n) - W(c_1, n)}_{\Delta^{redistribution}} + \underbrace{W(c_1, n) - W(c, n)}_{\Delta^{insurance}}.$$

Consumption allocation c_1 involves a consumption risk of the fixed-term contract of $\underline{\theta}$, but keeps the present value of consumption of each initial type at the same level as the original allocation c . Hence, $\Delta^{insurance}$ captures the welfare loss due to missing insurance within firm. Consumption allocation c_2 contains both the consumption risk and the change in transfers between initial types. The transition to fixed-term contract relaxes incentive-compatibility constraints between initial types for two reasons: the consumption of $\underline{\theta}$ is more volatile and income shifting is no longer possible. Thus, $\Delta^{redistribution}$ captures the welfare impact of the change in redistribution. Finally, (c', n') correspond to solving the standard government's problem given the contract allocation f' . Note that only at this stage the adjustment to labor supply are allowed. Hence, $\Delta^{efficiency}$ expresses the welfare gain from optimal adjustment of both consumption and labor along the incentive-compatibility constraints. For the details of this decomposition, see Definition 2.8 in the Appendix.¹⁹

Lemma 2.5. $\Delta^{insurance} \leq 0, \Delta^{efficiency} \geq 0$. $\Delta^{redistribution} \geq 0$ if $\underline{\theta} \in \arg \max_{\theta \in \Theta_1} \lambda(\theta)u'(c_1(\theta))$ and $\Delta^{redistribution} \leq 0$ if $\underline{\theta} \in \arg \min_{\theta \in \Theta_1} \lambda(\theta)u'(c_1(\theta))$.

Lemma 2.5 determines the signs of the decomposition terms: the switch from permanent to fixed-term contract leads to a utility loss due to lower insurance and utility gain due to labor adjustment. The sign of the redistribution component depends on the desired direction of redistribution. Fixed-term contract improves redistribution if assigned to a recipient of government transfers rather than a net taxpayer.

Lemma 2.5 suggests that there are two channels that may lead to the optimality of fixed-term contracts: redistribution and efficiency. I explore these channels in the two propositions below.

Assumption 2.1. *The distribution of productivity has full support: $\forall_{h \in \Theta^i} \mu(h) > 0$, and satisfies the first-order Markov property: for any $h \in \mathcal{H}$ such that $|h| > 2$ and any $s \in \mathcal{H}_{|h|-2}$ it is true that $\mu(h) = \mu(s, h_{|h|-1}, h)$.*

Proposition 2.3. *Suppose that (i) Assumption 2.1 holds and additionally the distribution of productivities is independent of the initial productivity draw, (ii) workers are risk neutral: $U(c, n) = c - v(n)$, (iii) $\underline{\theta}$ is the initial type with the lowest productivity and has a positive lifetime income:*

¹⁹Doligalski and Rojas (2016) use the similar decomposition of welfare change into redistribution and efficiency components in the static Mirrlees (1971) framework with an informal sector.

$\max_{s \in \mathcal{H}(\underline{\theta})} y(s) > 0$, (iv) the planner is Rawlsian: $\forall_{\theta \neq \underline{\theta}} \lambda(\theta) = 0$. Assigning fixed-term contract to type $\underline{\theta}$ is welfare improving.

To understand this proposition, consider a simple example with two initial types ($\Theta_1 = \{\underline{\theta}, \bar{\theta}\}$, $\bar{\theta} > \underline{\theta}$). The planner wants to maximize the utility of the low type and hence will redistribute from $\bar{\theta}$ to $\underline{\theta}$. What limits redistribution is ability of $\bar{\theta}$ to mimic the other type. If the low type has fixed-term contract, the mimicking is straightforward: $\bar{\theta}$ has to produce at each contingency as much as $\underline{\theta}$. If the low type has permanent contract instead, misreporting will also involve changing the allocation of output. $\bar{\theta}$ is more productive initially and hence will produce more in the first period. Then the firm pays him a part of the first period output in the future, allowing the mimicking worker to reduce the future labor supply. This income shifting implies that the high type gains more from misreporting when the low type has permanent contract rather than fixed-term contract.

The simplifying assumption of risk neutrality means that the planner cares only about the redistribution and not about the insurance ($\Delta^{insurance} = 0$). Since there is no utility loss from volatile consumption, the incentive constraints that prevent redistribution are relaxed only because of the prevented income shifting. With two initial types ($\Theta_1 = \{\underline{\theta}, \bar{\theta}\}$, $\bar{\theta} > \underline{\theta}$), we can express the redistribution gain explicitly as

$$\Delta^{redistribution} = \mu(\bar{\theta}) \sum_{s \in \mathcal{H}(\bar{\theta})} R^{1-|s|} \mu(s | \bar{\theta}) (v(\hat{n}(s)) - v(\tilde{n}(s))) > 0.$$

where \tilde{n} is labor allocation of the mimicking high type when $\underline{\theta}$ has permanent contract, while \hat{n} is the labor allocation when $\underline{\theta}$ has fixed-term contract. In line with the intuition above, the disutility from labor of the mimicking type is higher if the other type has fixed-term contract. Thus, $\Delta^{redistribution}$ is strictly positive. Since there is no utility loss due to missing insurance and $\Delta^{efficiency}$ is always non-negative, the overall welfare impact of switching contract of $\underline{\theta}$ is positive. Although the risk neutrality is a strong assumption, we can expect this result to hold also for moderate risk aversion, when $\Delta^{insurance}$ is sufficiently small.

Proposition 2.4. *Suppose that (i) Assumption 2.1 holds, (ii) there is some initial type $\underline{\theta}$ with permanent contract that supplies no labor: $\forall_{h \in \mathcal{H}(\underline{\theta})} n(h) = 0$ and faces downward labor distortions in future periods: $\exists_{h \in \mathcal{H}(\underline{\theta}) \setminus \underline{\theta}} s.t. \theta(h)u'(c(h)) > v'(0)$. Assigning fixed-term contract to type $\underline{\theta}$ is welfare improving.*

Proposition 2.4 shows that fixed-term contracts can improve the allocation of labor. Suppose that the distortions under permanent contracts are so severe that the some type has no lifetime earnings.²⁰ Notice that $\Delta^{insurance}$ is zero also in this case, although this time we do not impose

²⁰Optimum under permanent contracts has this feature when $v'(0) > 0$, $\mu(\underline{\theta})$ is sufficiently low and the planner wants to redistribute to $\underline{\theta}$.

risk neutrality. Since $\underline{\theta}$ does not supply labor, there is no need for volatile consumption. Moreover, $\Delta^{redistribution}$ is zero as well, for there is no scope for income shifting. The low type can gain only through the efficiency considerations.

Under permanent contracts, the planner discourages misreporting by reducing labor income of $\underline{\theta}$ at all histories. This is the case, because the output produced initially can be paid to the worker at any future history. Such income shifting is not possible with fixed-term contract. Under fixed-term contract the planner can lift some of the future distortions and generate additional resources, achieving $\Delta^{efficiency} > 0$. For instance, in the simplest iid case the classical ‘no distortion at the top’ result extended to the dynamic setting says that it is suboptimal to distort labor supply of the most productive type after any history. The planner should lift distortions of the most productive fixed-term workers. Note that not all distortions should be lifted, since they serve insurance purpose.

Hopenhayn and Rogerson (1993) claim that high severance payments cause labor misallocation. Their finding relies on the contractual friction: workers are paid a market wage in every period. Lazear (1990) shows that if instead labor contracts were flexible, the firm could design a compensation structure that nullifies the adverse effect of the firing cost on employment. In this paper the logic of Lazear (1990) holds: the high firing cost of permanent contract does not discourage firms from hiring workers.²¹ The firing cost does, on the other hand, encourage firms to offer a compensation structure that minimizes workers’ tax burden. The government can prevent firms from doing so either by introducing additional tax distortions or by promoting fixed-term contracts. Proposition 2.4 identifies the case when the latter is preferable.

2.5 Simple fiscal implementation

Dynamic optimal taxation literature suffers from very complicated tax implementations. I tackle this problem in my framework by considering a restricted taxation problem. The restricted problem is attractive for a few reasons. Its solution can be described with the well understood Saez (2001) formula from the static Mirrlees model. This solution can be implemented with a simple tax system, which, in the most favorable case, depends exclusively on current consumption expenditures. Furthermore, the restricted problem provides a tight lower bound on attainable welfare. In fact, in the next section I show numerically that for the typical social welfare functions the solution to the restricted problem coincides with the unrestricted optimum.

²¹In my framework there is no gain from workers switching jobs, as all firms are identical. However, even if there were efficient separations, high firing cost may still be efficient. Postel-Vinay and Turon (2014) show that firms facing high firing cost can persuade their workers to leave with a generous severance packages. Thanks to the high firing cost, the firm internalizes the worker’s utility loss from separation.

This section is structured as follows. First, I formalize the notion of the tax system and fiscal implementation. Then I consider the optimum with full consumption insurance: I define the restricted taxation problem and show that its solution can be implemented with simple tax system. Finally, I discuss the implementation of the dual labor market allocation.

2.5.1 The tax system

In the previous sections I characterized the optimal direct mechanism. This section is concerned with an indirect mechanism - a tax system. The tax can depend on all observables: history of labor income y , asset trades a and type of contract f as well as age t .

Definition 2.6. A *tax system* T is a collection of functions $T = \{T_t((y_k, a_k, f_k)_{k=1}^t)\}_{t=1}^{\bar{t}}$, where $T_t : (\mathbb{R} \times [-b, \infty) \times \{0, \bar{f}\})^t \rightarrow \mathbb{R}$.

We can define the set of equilibria corresponding to the tax system. Firms and workers, who take the tax system as given, optimize with respect to labor supply, savings, the type of labor contract as well as compensation structure. The tax system affects the equilibrium by modifying the budget constraint of workers.

Lemma 2.6. *The set of equilibria given the tax system T is*

$$\tilde{\mathcal{E}}(T) \equiv \arg \max_{\substack{c, n : \mathcal{H} \rightarrow \mathbb{R}_+ \\ y : \mathcal{H} \rightarrow \mathbb{R} \\ a : \mathcal{H} \rightarrow [-b, \infty) \\ f : \mathcal{H} \rightarrow \{0, \bar{f}\}}} \mathbb{E}U(c, n),$$

subject to the zero profit condition and the limited commitment constraints

$$\forall_{h \in \mathcal{H}_1} \mathbb{E}\pi_h(y, n) = 0,$$

$$\forall_{h \in \mathcal{H}} -f(h) \leq \mathbb{E}\pi_h(y, n) \leq 0,$$

the sequence of budget constraints

$$\forall_{h \in \mathcal{H}_1} c(h) = y(h) - a(h) - T_1(y(h), a(h), f(h)),$$

$$\forall_{h \in \mathcal{H} \setminus \mathcal{H}_1} c(h) = y(h) + Ra(h^{-1}) - a(h) - T_{|h|}\left((y(h^t), a(h^t), f(h^t))_{t=1}^{|h|}\right),$$

and no borrowing in the terminal period: $\forall_{h \in \mathcal{H}_{\bar{t}}} a(h) \geq 0$.

The tax system T *implements* the allocation (c, y, n) if there exist functions a and f such that $(c, y, n, a, f) \in \tilde{\mathcal{E}}(T)$.

2.5.2 The case of full consumption insurance

Suppose that it is optimal to assign permanent contracts and full consumption insurance to all workers. From Theorem 2.2 we know that the optimum under full commitment on the labor market provides a lower bound on welfare that can be achieved in the no commitment case. Furthermore, by Lemma 2.3 we know that without commitment the workers can no longer gain by deviating from truth-telling with mixed reporting strategies. Hence, I construct the restricted taxation problem by considering a full commitment problem, as in Theorem 2.1, with the incentive compatibility constraints that need to be satisfied only with respect to pure reporting strategies.

Definition 2.7. A *restricted taxation problem* is

$$\max_{(C(\theta), Y(\theta))_{\theta \in \Theta_1}} \sum_{\theta \in \Theta_1} \lambda(\theta) \mu(\theta) V_{\theta}(C(\theta), Y(\theta))$$

subject to the resource constraint

$$\sum_{\theta \in \Theta_1} \mu(\theta) (Y(\theta) - C(\theta)) \geq 0$$

and the incentive-compatibility constraints in pure strategies

$$\forall \theta, \theta' \in \Theta_1 \quad V_{\theta}(C(\theta), Y(\theta)) \geq V_{\theta}(C(\theta'), Y(\theta')). \quad (2.12)$$

Lemma 2.7. A *solution to the restricted taxation problem is implementable under permanent contracts.*

Lemma 2.7 means that the restricted taxation is a relevant lower bound for welfare in the unrestricted case, as there exists a direct mechanism that implements it. Note that the incentive constraints are tighter in the restricted problem than in the unrestricted problem. In the restricted problem the indirect utility function $V_{\theta}(C, Y)$ implicitly incorporates the optimality condition (2.8), which means that the marginal cost of output are equalized at every history. In the unrestricted case with permanent contracts the marginal cost of output needs to be only non-increasing over time. If it was increasing, the firm could shift the workers labor backwards in time, thereby reducing the overall disutility cost of labor. However, the marginal cost of output may be strictly decreasing over time, since the limited commitment of workers prevents the firm from shifting the labor forward. Recall that in this subsection we assume that the full consumption insurance for all workers is optimal. Then the solution to the restricted problem fails to reach optimum only if the optimal allocation features the decreasing marginal cost of output along some history. Such a distortion of labor supply may be optimal only if it relaxes the incentive compatibility constraints, which in turn happens only if the mimicking type prefers to produce later rather than earlier.

Suppose that the productivity process exhibits the mean reversion and the planner wants to redistribute from the initial low productivity type to the initial high productivity type. Then, by lifting the labor supply of the initial high type, the planner discourages the initial low type from misreporting. On the other hand, the planner cannot gain from a similar distortion while redistributing from the high to the low type, as lifting the initial labor supply of the low type would only encourage the misreporting of the high type. According to this intuition, we can expect the solution to the restricted problem to reach the unrestricted optimum when the planner wants to redistribute towards the less productive workers. This conjecture is true in the calibrated model considered in the next section.

The restricted problem is essentially the static [Mirrlees \(1971\)](#) model: the planner chooses lifetime consumption and lifetime income of each initial type subject to the initial incentive compatibility constraints in pure strategies. Similarly as in Section 2.3, let's denote by $T(Y)$ the net present value of taxes paid by an individual with a lifetime income Y : $T(Y(\theta)) \equiv Y(\theta) - C(\theta)$. Under the additional assumptions, we can express the solution to the restricted taxation problem with the modified [Saez \(2001\)](#) formula.

Assumption 2.2. Define $\hat{\mu}(\theta' | h_1) \equiv \sum_{h \in \mathcal{H}(h_1)} R^{1-|h|} \mu(h | h_1) / \bar{\beta} \mathbb{I}_{\theta(h)=\theta'}$, where \mathbb{I} is the indicator function. Take two initial types $h_1, s_1 \in \Theta_1$. If $s_1 > h_1$, then $\hat{\mu}(\theta' | s_1)$ first-order stochastically dominates $\hat{\mu}(\theta' | h_1)$.

Assumption 2.3. Θ_1 is an interval of real, non-negative numbers. The probability density function over Θ_1 is $f(\theta)$ and the cumulative distribution function is $F(\theta)$.

Proposition 2.5. Under Assumptions 2.2 and 2.3, if the implied lifetime income schedule $Y(\theta)$ is non-decreasing, the solution to the restricted taxation problem of an initial type $\theta \in \Theta_1$ satisfies

$$\forall \theta \in \Theta_1 \quad \frac{T'(Y(\theta))}{1 - T'(Y(\theta))} = \frac{1 - F(\theta)}{\theta f(\theta)} \frac{1 + \bar{\zeta}^u(\theta)}{\bar{\zeta}^c(\theta)} \mathbb{E} \left\{ (1 - \omega(\theta')) e^{\int_{\theta}^{\theta'} \frac{\bar{\xi}(\theta'')}{\bar{\zeta}^c(\theta'')} \frac{Y'(\theta'')}{Y(\theta'')} d\theta''} \middle| \theta' \geq \theta \right\}$$

where $\bar{\zeta}^c(\theta) = \sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta) \frac{\theta(h)n(h)}{Y(\theta)} \zeta^c(h)$ is the weighted lifetime average of the compensated elasticity of labor supply, $\bar{\xi}(\theta) = \bar{\beta}^{-1} \sum_{h \in \mathcal{H}(\theta_1)} R^{1-|h|} \mu(h | \theta_1) \xi(h)$ is the lifetime average wealth effect, $\bar{\zeta}^u(\theta) = \bar{\zeta}^c(\theta) + \bar{\xi}(\theta)$ is the lifetime average uncompensated elasticity of labor supply, $\omega(\theta) = \lambda(\theta) u'(C(\theta)/\bar{\beta})/\eta$ is the marginal social welfare weight of the initial type θ and η is the multiplier of the resource constraint.

Assumption 2.2 states that the distribution of productivity of initially higher types first-order stochastically dominates the distribution of productivity of the initially lower types. This assumption guarantees that the indirect utility function $V_{\theta}(C, Y)$ satisfies the Spence-Mirrlees single crossing property (see Lemma 2.11 in the Appendix). The single crossing property intuitively means that the initial higher type is more eager to work on average over the whole lifetime than

the initial lower type. If this property holds, any incentive-compatible lifetime income schedule is non-decreasing in the initial type. In order to apply the existing optimal tax formulas, we need to slightly modify the environment - Assumption 2.3 makes the initial distribution of types continuous. Given these assumptions, if the resulting income schedule is non-decreasing, we can express the optimal marginal tax rates with the formula derived by [Saez \(2001\)](#).

The tax rates depend on the distribution of types, the labor supply elasticities as well as social preferences. As the government is concerned only with redistribution, the marginal tax rates depend directly only on the initial distribution of types. Intuitively, if each worker had the same initial productivity, there would be no scope for the redistributive taxation - any inequality of income would be a matter of insurance. Furthermore, the elasticities that enter the tax formula are the lifetime averages. Specifically, the lifetime compensated elasticity of labor supply is an average compensated elasticity of labor supply over all histories weighted by output.

Fiscal implementation of the allocation $\{C(\theta), Y(\theta)\}_{\theta \in \Theta_1}$ in the static [Mirrlees \(1971\)](#) model is simple. It is enough to have a tax system that depends on labor income according to the function $T_y \equiv T \circ Y^{-1}$. In the dynamic setting agents make multiple choices, hence the tax system needs to prevent multidimensional deviations. Furthermore, insurance within firms requires backloading, which means that the labor income is not constant over the life-cycle. Implementing a full consumption insurance with a labor income tax would require a complicated, time-varying tax schedule. Instead, the tax can be based on consumption itself. Define *consumption expenditure* at the history h as the total income net of new savings: $x(h) \equiv y(h) + Ra(h^{-1}) - a(h)$. Consumption expenditure provides an attractive base for the redistributive tax since it is observable by the tax authority and stable over the workers' lifetime.

Take an allocation of lifetime consumption and labor income $\{C(\theta), Y(\theta)\}_{\theta \in \Theta_1}$, where the lifetime income is increasing in the initial type. Consumption expenditure of type θ is $x(\theta) \equiv Y(\theta)/\bar{\beta}$. Define a *consumption expenditure tax* as $\bar{T}_x \equiv \bar{T} \circ x^{-1}$, where $\bar{T}(\theta) \equiv (\bar{Y}(\theta) - C(\theta))/\bar{\beta}$ is the average tax paid by initial type θ . Extend this function to the non-negative real half-line with T_x , which is equal to \bar{T}_x for values of x assigned for some type, and otherwise takes a prohibitively high value.

Theorem 2.4. *Take any allocation (c, y, n) and the corresponding allocation of lifetime consumption and income $\{C(\theta), Y(\theta)\}_{\theta \in \Theta_1}$ that is consistent with incentive compatibility constraints (2.12). Suppose that the borrowing limit is sufficiently high: $b \geq -\min_{h \in \mathcal{H}} \left\{ \sum_{t=1}^{|h|} R^{1-|h|} y(h^t) - Y(h_1) \right\}$. If \bar{T}_x is convex, then the allocation can be implemented with the tax system*

$$\forall_{t=\{1 \dots \bar{t}\}} T_t((y_k, a_k, f_k)_{k=1}^t) = T_x(x_t), \text{ where } x_t \equiv y_t + Ra_{t-1} - a_t.$$

If \bar{T}_x is not convex, the allocation can be implemented with the tax system

$$\forall_{t=\{1\ldots\bar{t}\}} T_t((y_k, a_k, f_k)_{k=1}^t) = T_x(x_t) + \alpha(x_t - x_1)^2,$$

where α is high enough such that $\bar{T}_x(x) + \alpha(x - x_1)^2$ is convex in x .

In the simplest case all we need for fiscal implementation is the time-invariant redistributive tax schedule based on current consumption expenditures.²² Note that when the consumption expenditure tax is locally regressive (i.e. \bar{T}_x is not convex), we need to add a corrective term that discourages variation in expenditures. Although the limited commitment prevents wage randomization, workers still can introduce a variation in expenditures over time. When a tax is regressive, such fluctuations would reduce their average tax burden and hence would attract workers with sufficiently low risk aversion. The corrective term convexifies the tax system and prevents this type of tax avoidance.²³ Conversely, when the consumption expenditure tax is progressive, no history dependence is required.

Corollary 7. *Consider Theorem 2.4. Suppose the borrowing is insufficient:*

$$b < -\min_{h \in \mathcal{H}} \left\{ \sum_{t=1}^{|h|} R^{1-|h|} y(h^t) - Y(h_1) \right\}.$$

The allocation can be implemented with the tax system as in Theorem 2.4 combined with the governmental lending to workers.

In this setting nothing prevents the government from lending to workers. As the debt repayment is contingent only on time, the government can always enforce repayment with taxes. Hence, the government in this setting can always relax the borrowing constraint of workers enough for Theorem 2.4 to hold. [Cole and Kocherlakota \(2001\)](#) found the similar relaxation of the borrowing limits to be optimal in the hidden income model with private storage.

[Kocherlakota \(2005\)](#) showed that NDPF can be implemented with the labor income tax that depends on the whole history of labor income and capital income tax which depends on current and previous labor income. [Albanesi and Sleet \(2006\)](#) provide a simpler implementation in the environment with independently and identically distributed productivity shocks, in which the tax depends

²²It is recognized in the literature that a non-linear consumption tax is difficult to implement if the government does not observe each individual transaction. However, the tax I propose does not differentiate between different consumption goods. Hence, the government can simply base the non-linear tax on the total income net of new savings, which by the budget constraint equals consumption expenditures. Note that the tax code in US has this feature, as the capital gains are taxed only when they are realized, i.e. when they cease to be savings.

²³Any schedule that is convex in x_t and equal to $T_x(x_1)$ for $x_t = x_1$ would work. For instance, workers in the period $t > 1$ can face a linear tax schedule or, in the simplest but perhaps the least realistic case, a tax that depends only on x_1 and not on current expenditures x_t .

jointly of current income and assets. In these implementations taxes are allowed to vary with time period, or equivalently age of a worker. By Theorem 2.4 the fiscal implementation of the optimum can be made still simpler with permanent contracts. The tax schedule is time invariant, depends only on current total income net of new savings (with the possible correction term if the tax schedule is regressive) and no additional tax on capital is required. This result holds irrespective of the persistence of productivity shocks. Although assigning permanent contracts to all workers is not always optimal, it is the feature of the utilitarian optimum in the calibrated model I consider in Section 2.7.

Existing tax codes bear a similarity to the consumption expenditure tax, in which savings are taxed only when they are spent on consumption. Unrealized capital gains are not taxed in US - income from the increased value of stocks is taxed only when the stocks are sold. [Le Maire and Schjerning \(2013\)](#) describe the Danish income tax for self-employed, which allows to retain earnings within the firm and pay them later in order to smooth the tax payments. Progressive consumption expenditure tax has been advocated by [Bradford \(2000\)](#). A flat tax proposed by [Hall and Rabushka \(1995\)](#) is a special case of such tax.²⁴

2.5.3 The case of a dual labor market

When the dual labor market is optimal, the fiscal implementation is more complicated, as the tax system needs to insure the fixed-term workers against the productivity shocks. The tax system needs to separate the initial fixed-term workers and the initial permanent workers, hence the dependence of the tax system on the contract type may be required.²⁵ Nevertheless, the restricted taxation problem described in the previous subsection is still useful to describe the part of the tax system faced by the initial permanent workers. If none of fixed-term workers is tempted to mimic the initial permanent workers,²⁶ the restricted problem provides an implementable lower bound on welfare of the initial permanent workers. We only need to modify the resource constraint in order to capture the resource cost of transfers to the fixed-term workers.

The tax system of the fixed-term workers will follow findings of the NDPF literature. Specifically, it will involve savings taxation.

Proposition 2.6. *Take any allocation (c, y, n) that is implemented by some direct mechanism and involves dual labor market, where consumption of fixed-term workers is bounded away from zero. Fiscal implementation requires taxing assets of fixed-term workers.*

²⁴[Bradford \(2000\)](#) and [Hall and Rabushka \(1995\)](#) support the consumption tax because it encourages savings while allowing for redistribution. In my model savings play no productive role, since there is no capital. Instead, asset trades alleviate the contracting friction within the firm.

²⁵Notice that fixed-term workers may be converted to permanent workers at some histories. The tax schedule they should then face after conversion is likely to be different than the tax schedule of the initial permanent workers. Hence, it is not enough that the tax depends only on a current contract type.

²⁶As is likely to be the case, since fixed-term contracts facilitates redistribution toward the workers that have them.

This results is an implication of the inverse Euler equation, which holds in this environment, and the volatile consumption of fixed-term workers. By discouraging savings, capital taxation helps to provide incentives for hard work in the next periods. Following [Golosov and Tsyvinski \(2006\)](#) we can interpret this result as public insurance program with assets testing.

2.6 Empirical evidence

The model yields testable implications about the income risk of labor contracts with different firing cost. In this section I use the administrative data of employment spells in Italy to show that indeed fixed-term contracts coincide with a higher residual variance of income (conditional on continuous employment) and that this difference is economically significant. In a related study [Guiso, Pistaferri, and Schivardi \(2005\)](#) show that Italian firms insure their workers, but they do not differentiate between different types of contracts. [Lagakos and Ordonez \(2011\)](#) document that high-skilled workers receive more insurance within the firm than low-skilled in US. It is consistent with the evidence provided by [Bishow and Parsons \(2004\)](#) that white collar workers are more frequently offered severance pay than blue collar workers.

2.6.1 Labor contracts in Italy

Italy in its modern history experienced a proliferation of distinct labor contracts.²⁷ I focus on only two types of contract: permanent (*il contratto a tempo indeterminato*) and fixed-term (*il contratto a tempo determinato*). Prior to the reforms in 2014 permanent contract used to feature exceptionally high firing cost. An employer could legally dismiss permanent workers for two reasons: difficult situation of the firm or inadequate fulfillment of tasks by the worker. Any fired worker could sue the company for an unfair dismissal. If the judge decides that dismissal was unfair, the worker had a right to be rehired by the original firm and compensated for the income lost during the legal process.²⁸ [Ichino, Polo, and Rettore \(2003\)](#) provide evidence that judges decision are not impartial: judges were less likely to find a dismissal justified when the unemployment was high. [Flabbi and Ichino \(2001\)](#) suggests that high firing cost leads to very low turnover rate in large Italian service companies.

Fixed-term contracts do not allow for worker's dismissal justified by a difficult situation of the company. However, as the contract expires, the firm may decide not to extend the contract and

²⁷[Tealdi \(2011\)](#), who provides an overview of labor reforms in Italy, state that in 2006 there were 46 different labor contracts.

²⁸As [Ichino, Polo, and Rettore \(2003\)](#) put it, "...firing costs are higher in Italy than anywhere else, because this is the only country in which, if firing is not sustained by a just cause (...), the firm is always forced to take back the employee on payroll and to pay the full wage that he/she has lost during the litigation period plus welfare contributions; in addition, the firm has to pay a fine to the social security system for the delayed payment of welfare contributions up to 200 percent of the original amount due."

hence terminate the employment relationship at no cost.²⁹ I conclude that permanent and fixed-term contracts in Italy are close empirical counterparts of permanent and fixed-term contracts as described in the theoretical framework.

2.6.2 Empirical model

I measure the lack of insurance residually, as a variation in income which cannot be explained by fixed personal characteristics, age, tenure, labor market experience, firm type, sector, location or time effects. Consider the following model:

$$\log(y_{ijt}) = \rho + W'_{it}\alpha + F'_{jt}\beta + M'_{ijt}\gamma + D'_t\delta + \epsilon_{ijt}, \quad (2.13)$$

where W_{it} includes worker's time invariant and time varying characteristics, F_{jt} includes firm's time invariant and time varying characteristics, M_{ijt} includes match characteristics such as tenure and type of contract, D_t are yearly fixed effects and ϵ_{ijt} is the error term. The parameter of interest is the variance of ϵ_{ijt} , which captures the residual income risk, conditional on being continuously employed. Let's compute the difference of (2.13)

$$\log\left(\frac{y_{ijt}}{y_{ijt-1}}\right) = \Delta W'_{it}\alpha + \Delta F'_{jt}\beta + \Delta M'_{ijt}\gamma + \Delta D'_t\delta + \Delta\epsilon_{ijt} \quad (2.14)$$

Take a vector of variables $X \in \{W, F, M\}$ and denote its vector of parameters by ξ . Divide X into three components:

$$X_{ijt} = [X^1_{ij}, X^2_{ijt}, X^3_{ijt}],$$

where X^1_{ij} involves variables which are fixed in time, X^2_{ijt} variables that depend linearly on year, such as age, labor market experience or tenure, and X^3_{ijt} are variables that depend on time non-linearly. Let's separate the vector of parameters ξ in the same way into ξ_1 , ξ_2 and ξ_3 . Then we can write

$$\Delta X'_{ijt}\xi = \sum \xi_2 + \Delta X'^3_{it}\xi_3,$$

and equation (2.14) becomes

$$\log\left(\frac{y_{ijt}}{y_{ijt-1}}\right) = \sum \alpha_2 + \sum \beta_2 + \Delta W'^3_{it}\alpha_3 + \Delta F'^3_{jt}\beta_3 + \Delta M'^3_{ijt}\gamma_3 + \Delta D'_t\delta + \epsilon_{ijt},$$

²⁹In the period I consider the firm could extend fixed-term contract once. The second extension lead to the automatic conversion of the contract into a permanent one. Labor reforms in 2014 allowed to up to 5 extensions that together with the original contract last no longer than 3 years.

where $\varepsilon_{ijt} = \Delta\epsilon_{ijt}$. In this way we avoid the need to estimate the fixed effects of workers, firms and a match, which greatly reduced the number of parameters. Furthermore, this specification is robust to possible correlation between individual fixed effects and tenure or labor market experience.³⁰

How does the variance of the error term in (2.13) depend on a type of employment contract? Assume that the distribution of error ϵ_{ijt} is independent of time and denote the variance of error with permanent contract by σ_P^2 and the variance of error with fixed-term contract by σ_{FT}^2 . Let's call $\frac{\sigma_{FT}^2}{\sigma_P^2}$ a *risk ratio*. The risk ratio greater than 1 means that fixed-term contracts imply more income risk, or equivalently less income insurance, than permanent contracts. The risk ratio is equal

$$\frac{\sigma_{FT}^2}{\sigma_P^2} = \frac{1 - \rho_P}{1 - \rho_{FT}} \frac{Var(\varepsilon^{FT})}{Var(\varepsilon^P)}$$

where ρ_x is the autocorrelation of errors when contract is $x \in \{P, FT\}$. If errors for two contract types have the same autocorrelation, then the risk ratio is simply given by the ratio of variances of errors from the differenced equation (2.14).

2.6.3 Data

The data comes from *Work Histories Italian Panel* (WHIP), a sample of administrative records of Italian employment histories.³¹ The time-span in which permanent and fixed-term contracts can be observed separately is 1997-2004. The data is at the annual frequency. I consider only a full time jobs and annualize the real income from a given job by dividing it by an average number of working days.

I extract all two-period employment spells of a given individual at the given firm with a contract of a given type. As an illustration of this procedure, consider the following example of a work history.

Table 2.1: An example of an employment history

year	company	contract
1998	A	fixed-term
1999	A	fixed-term
2000	B	fixed-term
2001	B	permanent
2002	B	permanent
2003	B	permanent

A worker with such an employment history was working on a fixed term contract for company *A* for

³⁰See discussion in Guiso, Pistaferri, and Schivardi (2013).

³¹Work Histories Italian Panel is a database of work histories developed thanks to the agreement between INPS and University of Turin. For more information, see <http://www.laboratoriorevelli.it/whip>.

two years. Then the worker moved to a company B for one year of fixed-term employment followed by the permanent employment. From this employment history three two-period employment spells are extracted: 1998 : 1999 at the company A with a fixed term contract and 2001 : 2002, 2002 : 2003 at the company B with permanent contract. I do not use the spell 1999 : 2000, as it involved a change of an employer, nor the spell 2000 : 2001, as it involved a change of contract.

For each 2-period employment spell, the logarithm of ratio of annualized income is computed. I remove outliers separately for two types of contract by considering only the spells with $\log\left(\frac{y_{ijt}}{y_{ijt-1}}\right)$ within three standard deviations from the sample mean. The explanatory variables used are: worker characteristics (gender, geographical region), firm characteristics (firm's age, sector), match characteristics (tenure, type of job) as well as annual dummies.

2.6.4 Results

Equation (2.14) is estimated with OLS separately for each type of contract.³² Then I take squared residuals from both regressions, pool them into a one vector and regress them on a set of explanatory variables that includes a 'fixed-term contract' dummy variable. This procedure is essentially the [White \(1980\)](#) test for heteroskedasticity of the error term. A significant positive estimate of the parameter of the 'fixed-term contract' dummy means that fixed-term contracts are associated with higher variance of errors from the difference equation (2.14). The main results of this regression are reported in Table 2.2, the full results and auxiliary estimates are reported in Appendix 2.8.

Table 2.2: Regression of $\hat{\varepsilon}_t^2$ (main estimates)

variable	coefficient	t	95% confidence interval
constant	0.0347***	10.557	(0.028, 0.041)
fixed-term contract	0.009***	13.058	(0.008, 0.01)
$\log(y_{ijt})$	-0.0019***	-5.591	(-0.003, -0.001)

*** - statistically significant at the 1% level.

The fixed-term dummy is positive and highly significant, which means that the variance of errors of the auxiliary differenced regression are higher for fixed-term contracts: $Var(\varepsilon^{FT}) > Var(\varepsilon^P)$. Since variance of errors vary with other characteristics as well (such as log income, as reported in Table 2.2), in order to compute the lower bound on the risk ratio, consider a male worker from north-west of Italy in 1998, who starts a job in services at the median income ($\approx 20,000$ euros). In this case $Var(\varepsilon^{FT})/Var(\varepsilon^P) = 1.78$.

I use similar method to examine the impact of the type of contract on the autocorrelation of errors. The product of the lagged and current residuals is regressed on a set of explanatory variables

³²There are 179,831 two-period spells with permanent contract and 3,486 with fixed-term contract.

Table 2.3: Regression of $\hat{\varepsilon}_{t-1}\hat{\varepsilon}_t$ (main estimates)

variable	coefficient	t	95% confidence interval
constant	0.0031	1.383	(−0.001, 0.008)
fixed-term contract	−0.0012	−1.568	(−0.003, 0.001)
$\log(y_{ijt})$	−0.0006*	−2.742	(−0.003, −0.001)

* - statistically significant at the 10% level.

and a ‘fixed-term contract’ dummy. In this case the impact of fixed-term contract is statistically insignificant at 10% level (see Table 2.3). Moreover, using the point estimate for a median male worker as before, we arrive at the correlation ratio $\frac{1-\rho^P}{1-\rho^{FT}} = 0.9997$. Hence, the risk ratio is very well approximated by the ratio of variances of error from the differenced equation

$$\frac{\sigma_{FT}^2}{\sigma_P^2} \approx \frac{Var(\varepsilon^{FT})}{Var(\varepsilon^P)} = 1.78.$$

The income risk faced by the median worker with fixed-term contract is 78% higher than the income risk faced by the similar worker with permanent contract. It is an economically significant value. A worker with permanent contract earning a median income can expect that with 95% probability his next year income will be between 17,509 and 23,777 euros. The same worker with a fixed-term contract will have a wider confidence interval of 16,522 to 24,927 euros.

The analysis above may suffer from a selection problem. That would be the case if firms offering more risky jobs used fixed-term contracts, while more stable firms hire on a permanent basis. A proper causal analysis of relation between a type of contract and the residual volatility of income is an interesting topic for future research.

2.7 Quantitative exercise

In this section I calibrate the simple life-cycle model using the Italian data (WHIP) and describe the set of constrained efficient allocations.

2.7.1 Calibration

The sample is divided into two age groups: young (below the median age) and old (above or equal to the median age). Only wage workers with a full-time job are considered. With the data at hand the persistence of income on such a long time period is not observed - at most 8 years of income for each individual are available. Rather than assuming the earning process that is independent across time,

I use the data on total employment spell with a given employer. Within each age group, I divide workers between permanently employed (with permanent employment contract and sufficiently long total employment spell at the current employer) and temporarily employed (fixed-term workers and workers with shorter total employment spell). I assume that income of permanently employed old is informative about the future income of permanently employed young³³. Another rationale for this division is that, according to the theory, the data on labor income is more informative of productivity for workers that are not engaged in long-term relationship with their employers. As it turns out, for both types of workers at each history the income is strictly increasing in age (see figure 2.1). Under assumption of no borrowing in the data, the income process is informative about the productivity for all age/contract type groups.³⁴

I take the mean labor income of young within each contract group and assign probability of each contract group by relative frequency in the data. The earnings distribution of old workers is described with the Gaussian mixture model. The Gaussian mixtures can approximate well complex distributions (Marin, Mengersen, and Robert (2005)) and were successfully used to capture higher moments of the US earnings distribution (Guvenen, Karahan, Ozkan, and Song (2015)). I estimate the mixture by maximum likelihood Expectation-Maximization algorithm of Dempster, Laird, and Rubin (1977). Then, in order to keep the model simple, I take the estimated means of each component of the mixture as a distinct earning realization that occurs with the probability equal to the weight of this component in the mixture. In practice, for both groups of old workers (permanently and temporarily employed) the mixture of two normal distributions fits the data well. Figure 2.1 presents the estimation results. Income is reported in euros per year at the 2004 prices.

I use logarithmic utility from consumption and iso-elastic disutility from labor with compensated elasticity of 1:

$$u(c) - v(n) = \log(c) - \Gamma \frac{n^2}{2}.$$

There are 7 parameters left to determine: the productivity at each history and the labor disutility parameter Γ . The productivities are pinned down with the first-order condition of labor supply

$$\theta = y(\theta) \sqrt{\Gamma \frac{1 - T(y(\theta))/y(\theta)}{1 - T'(y(\theta))}}, \quad (2.15)$$

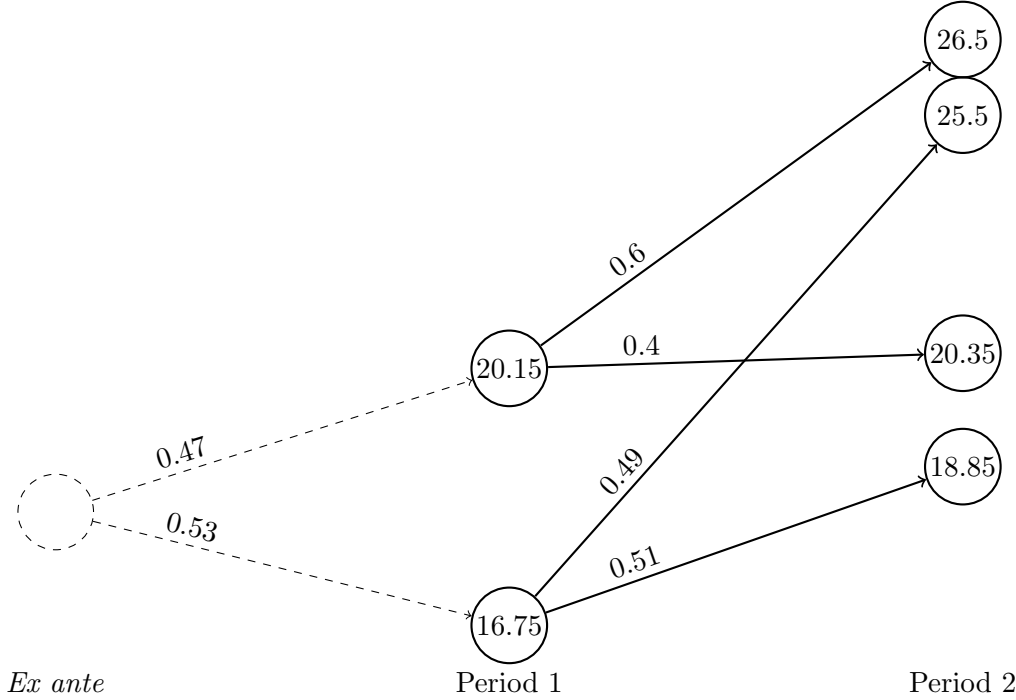
where $T(y)$ is the actual Italian tax schedule.³⁵ Without the loss of generality Γ is set to 1 - by

³³In fact, in the dataset some permanent workers cross the threshold between age groups.

³⁴Since I consider only two periods, the upward time trend dominates the stochastic variation. In the future work I plan to estimate the model for more age groups, where the issue of disentangling current output and insurance is likely to emerge.

³⁵Italy undertook a series of tax reforms in the considered period. I use the tax schedule from year 2000, which captures the average shape of the tax function in these years.

Figure 2.1: The estimated income process [1,000 EUR per year]



the first order condition (2.15) varying Γ would simply rescale all the productivities. The discount factor β is equal to 0.5, corresponding to the period of 17 years.

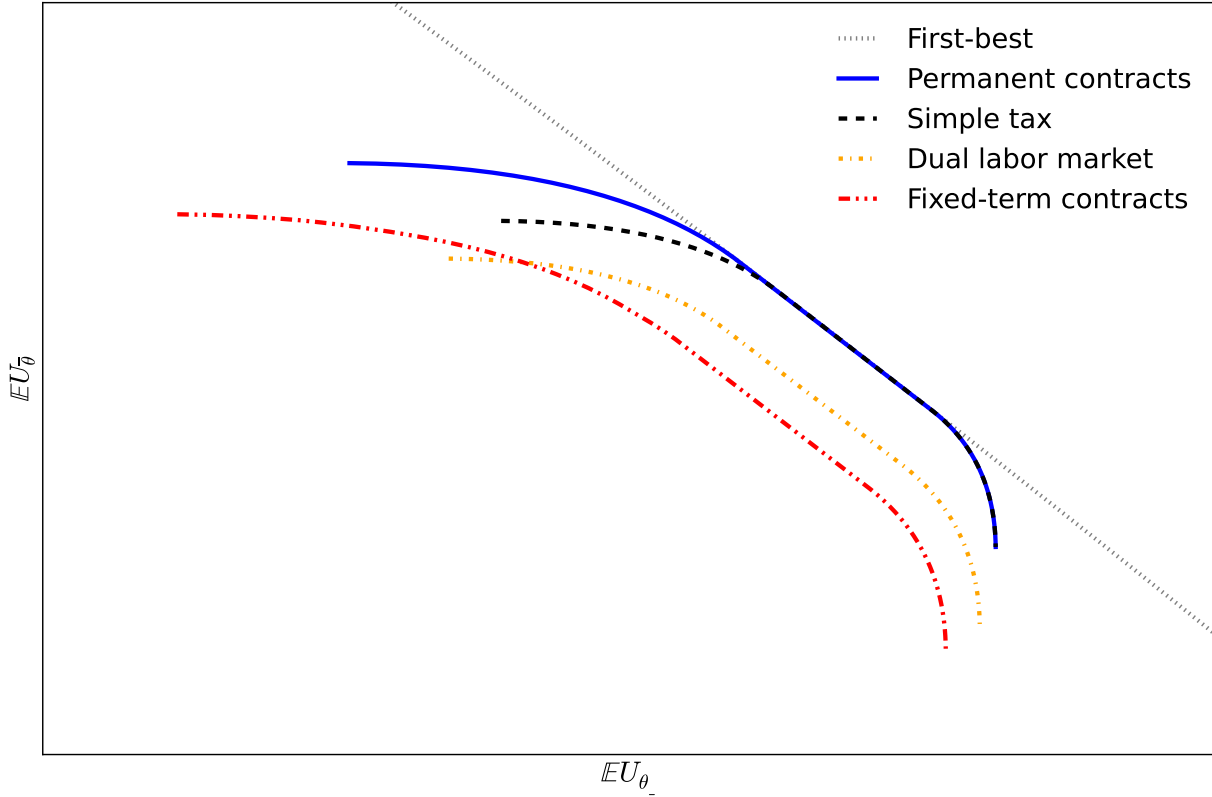
2.7.2 Pareto Frontiers

Figure 2.2 shows Pareto frontiers of four different regimes. ‘Fixed-term contracts’ regime corresponds to the NDPF economy, in which all workers receive fixed-term contracts and firms do not provide insurance. ‘Dual labor market’ frontier describes the economy in which the initially more productive type is employed permanently, while the other is employed on a fixed-term basis.³⁶ ‘Permanent contracts’ regime is characterized by both initial types receiving permanent employment. Finally, the ‘Simple tax’ regime corresponds to the restricted taxation problem considered in Section 2.5. In this regime all workers have permanent contracts and the allocation can be implemented with a simple consumption expenditure tax described by Theorem 2.4. I plot as well the Pareto frontier of the first-best allocation as an indicator of what is feasible if we abstract from incentive issues. The first-best is characterized by the full consumption insurance and efficient allocation of labor at each history. In each regime the government raises the same net tax revenue as the actual

³⁶I do not plot the Pareto frontier of the other configuration of contracts, in which the initially low productivity type receives permanent contract and the initially high type receives fixed-term contract. It is dominated by the permanent contracts case.

Italian tax schedule.

Figure 2.2: Pareto frontiers in the calibrated economy



Whenever the worker is employed on fixed-term contract, the only source of insurance against the productivity risk is the tax system. Information constraints prevent the government from implementing simultaneously full consumption insurance and efficient allocation of labor. As a result, the Pareto frontier of any regime in which some workers are employed on a fixed-term basis is bounded away from the first-best Pareto frontier. On the other hand, permanent contracts allow for coexistence of full insurance and efficient labor supply if the redistribution between initial types is limited. The permanent contracts frontier coincides with the first-best when the social preferences are not strongly redistributive.

By Theorem 2.3 we know that the regime with only fixed-term contracts is Pareto dominated by a regime in which at least one type of worker has permanent contract. Figure 2.2 shows that for the calibrated parameter values it is always optimal to assign permanent contracts to all workers. Furthermore, in any constrained efficient allocation all workers enjoy full consumption insurance. The dual labor market regime improves upon the fixed-term regime when the planner cares predominantly about the initially less productive workers, but the gains from assigning permanent

Table 2.1: Optimal allocations for different social welfare functions

Social preferences:	utilitarian	libertarian	Rawlsian	anti-Rawlsian
Welfare (cons. equiv.)				
Laissez-faire	100%	100%	100%	100%
Fixed-term contracts (NDPF)	102.8%	102.7%	107.2%	105.9%
Dual labor market	103.4%	103.3%	108.1%	104.8%
Simple tax	104.3%	104%	108.3%	105.8%
Permanent contracts (optimum)	104.3%	104%	108.3%	107.2%
Relative gain from permanent contracts	53.3%	49.3%	12.9%	20.3%

Note: The libertarian planner maximizes the average utility subject to no redistribution between the initial types. Taxation plays only an insurance role and workers would voluntarily decide to participate in a public insurance scheme. The anti-Rawlsian planner is the opposite of the Rawlsian planner and maximizes the utility of the most well-off type.

contract to both types are even greater. The welfare gap for the Rawlsian planner between the permanent contracts and the dual labor market regimes is 0.6% in consumption equivalent terms.

The simple tax allocation coincides with ‘Permanent contracts’ regime unless the planner strongly favors the initial high type. Labor distortions which are possible under permanent contracts, but not in the simple tax regime, are useful only when the planner wants to redistribute from the bottom to the top. In all the other cases, the constrained efficient allocations can be implemented with the consumption expenditure tax.

Table 2.1 compares regimes in terms of welfare under different social welfare functions. Utilitarian planner maximizes the expected utility of workers. Libertarian planner maximizes the expected utility of workers subject to the restriction of no transfers between initial types. The Rawlsian maximizes the utility of the least well-off worker, while the anti-Rawlsian planner cares about the most well-off worker. The benchmark allocation is a laissez-faire, which involve fixed-term contracts, no public insurance and uniform lump-sum taxation to cover the government expenditures. Allocations are compared using the consumption equivalent measure: by which factor we need to increase consumption of workers at each history in the laissez-faire allocation to obtain the same welfare as in the considered allocation. When we consider the less redistributive social planners (utilitarian and libertarian), the NDPF captures close to two-thirds of the gains from constrained efficient allocation. It means that the simpler tax that encourages firms’ insurance improves upon the complicated tax system prescribed by NDPF by close to 50% in relative terms (the last row of Table 2.1). The relative welfare gain from using permanent contracts in comparison to fixed-term contracts is smaller for social preferences focused on redistribution.

Consider the alternative economy in which the initial differences between two types are greater than in Italy: suppose that the initial productivity of less productive type is lower by 10%. The corresponding Pareto frontiers are shown on Figure 2.3. Now the Rawlsian planner prefers the dual labor market regime. The welfare gain of the dual labor market over the permanent contract

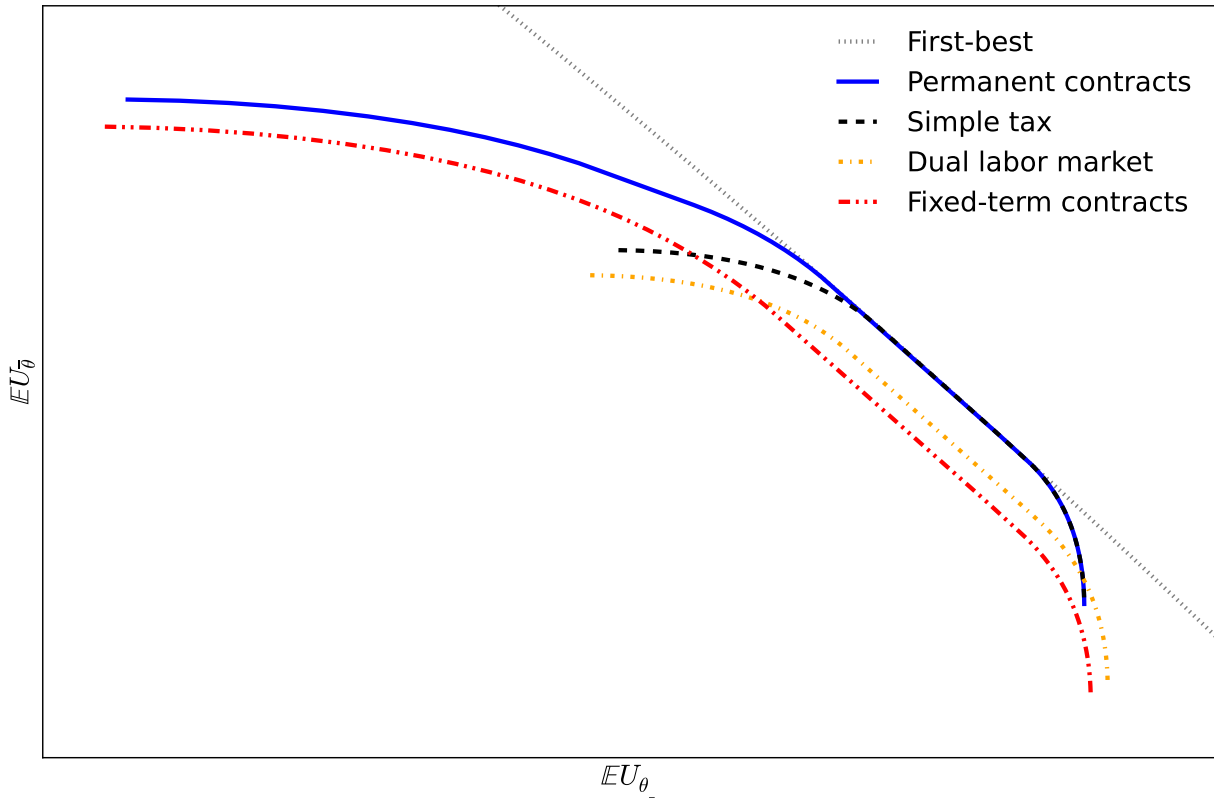
Table 2.2: Welfare impact of a dual labor market

	$\Delta^{insurance}$	$\Delta^{redistribution}$	$\Delta^{efficiency}$	total change
Original economy	-2.7%	2%	0.4%	-0.3%
Alternative economy	-2.8%	3%	0.7%	1%

Note: The decomposition is formally stated in Definition 2.8 in Appendix.

regime is significant: 1% in consumption equivalent terms.³⁷ Table 2.2 decomposes the welfare gain from a dual labor market into three components, corresponding to the change in consumption insurance, redistribution and efficiency. The main difference between the original economy and the alternative economy with increased initial differences is a greater gain in redistribution. The increased difference in initial productivities means that the deviating high type benefits more from income shifting: producing more in the initial period and getting paid in the second period. Income shifting is possible under permanent contracts and is prevented by the dual labor market.

Figure 2.3: Pareto frontiers in the alternative economy with increased initial differences



³⁷The dual labor market and the permanent contract regime yield roughly the same welfare when the initial low productivity is lower by 4% in comparison to the calibrated value.

2.8 Conclusions

Firms are the natural insurers of their employees. First, competition in the labor market gives firms strong incentives to shelter workers from risks. Second, companies arguably have the best knowledge of their workers' productivity (with an exception of workers themselves). The insurance role of firms should be acknowledged by the optimal tax theory, as it can resolve its shortcoming: the excessive complexity. In this paper I show that incorporating firms into the dynamic taxation framework leads to a fully optimal tax system that is simple and realistic. The government optimally outsources insurance to firms by promoting permanent employment contracts. The only remaining role for the government is redistribution, which can be conducted with simple instruments. In a calibrated model of Italy all constrained efficient allocations which do not involve redistributing from the poor to the rich can be implemented with a comprehensible tax system: a time-invariant tax schedule that depends exclusively on current consumption expenditures.

Empowering the private sector to insure workers comes at a price. Firms insure their workers by shifting income from the times of high to the times of low productivity. However, this intertemporal reallocation can be used to avoid taxes in the following way: a productive worker shifts the income to the future and collects income support today. A redistributive government can limit such a behavior without reducing the generosity of transfers by promoting fixed-term contracts at low levels of earnings. This redistributive argument provides a novel perspective on dual labor markets in which permanent and fixed-term contracts coexist.

The analysis could be extended in several directions. The focus of this paper is on workers' heterogeneity. Introduction of heterogeneous production opportunities for firms would allow for analysis of temporary jobs, which in fact are the primary reason for existence of fixed-term contracts. Careful treatment of employees' outside option is vital to understanding insurance within firm. Specifically, the limited commitment of workers may be muted by search and mobility costs. Finally, in this paper there is no moral hazard problem within firms. It is worth examining how the optimal redistributive tax system interacts with the incentive provision in the private sector.

Appendix

Proofs and auxiliary lemmas

Proofs from Section 2.2

Proof of Lemma 2.1. Take (σ, n) such that at some history $h \in \mathcal{H}$ the firm has positive expected profits. A competitor could profitably steal the worker by offering (σ', n') , where $\sigma' = \sigma$ and for all $r \in \mathcal{R}$ $n'_r(h) = n_r(h) - \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E} \pi_h(c \circ r, n) / 2\theta(h)$ and $n'_r(s) = n_r(s)$ for any other history $s \neq h$. This offer yields half of the profits of (σ, n) and would be preferred by the worker, as it involve less labor supply. Suppose instead that profits are negative and lower than $f(r(h))$. Then the firm would prefer to fire the worker and incur the firing cost rather than to keep the worker. Hence, (σ, n) has to satisfy $\forall_{h \in \mathcal{H}} f(r(h)) \leq \sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E} \pi_h(c \circ r, n) \leq 0$. \square

Proof of Lemma 2.2. The firm cannot expect losses initially, as she could offer a contract that yields zero profits by equalizing worker's output to worker's income at each history. Together with the limited commitment constraint at the initial type (2.3) it yields the zero profit condition (2.4).

Suppose there is an equilibrium (σ, n) which does not belong to $\mathcal{E}(c, y, f)$. It means there is another contract (σ', n') which yields strictly greater expected utility to the worker subject to the zero profit condition and limited commitment constraints. Define $\sigma'' = \sigma'$ and $\forall_{r \in \mathcal{R}} \forall_{h \in \mathcal{H}} n''_r(h) = n'_r(h) + \epsilon$, where $\epsilon > 0$. For epsilon sufficiently small (σ'', n'') yields positive profits and greater expected utility than (σ, n) . Hence, (σ, n) cannot be an equilibrium.

Suppose that $(\sigma, n) \in \mathcal{E}(c, y, f)$ is not an equilibrium. It means that there is another (σ', n') that yields positive profits and the expected utility greater than (σ, n) . This in turn implies that there is yet another contract which yields zero profits at each history and the expected utility greater than (σ', n') . It contradicts the fact that $(\sigma, n) \in \mathcal{E}(c, y, f)$. \square

Proofs from Section 2.3

Lemma 2.8. *Under full commitment on the labor market, all agents optimally enjoy full consumption insurance.*

Proof of Lemma 2.8. Take any allocation (c, y, n) which is incentive-compatible (i.e. (n, σ^*) is the equilibrium, given (c, y)) and do not involve full consumption insurance. For each type $h \in \mathcal{H}$ find a full-insurance consumption level $\bar{c}(h)$ with equality $\sum_{t=1}^{\bar{t}} \beta^{t-1} u(\bar{c}(h)) = \sum_{s \in \mathcal{H}(h_1)} \beta^{1-|s|} \mu(s | h_1) u(c(s))$. Set $\bar{y}(h)$ to the average of histories of this length: $\sum_{s \in \mathcal{H}_{|h|}(h_1)} \mu(s | h_1) y(s)$. This way both the lifetime consumption and labor income are deterministic functions of the initial type report. As the worker receives more consumption insurance, the planner frees some resources.

Before we prove that truth-telling is the equilibrium strategy given the new outcomes, let's define a useful class of reporting strategies. Take some pure reporting strategy $r \in \mathcal{R}$. The *statistical mimicking* strategy σ_r^{stat} is a mixed reporting strategy such that

$$\forall_{s_1 \in \Theta_1} \forall_{h \in \mathcal{H}} \mu(h | r(s_1)) = \sum_{r' \in \mathcal{R}} \sigma_r^{stat}(r') \sum_{h' \in \mathcal{H}} \mu(h' | s_1) \mathbb{I}_{r'(h')=h},$$

where \mathbb{I} is an indicator function. Statistical mimicking strategy generates the distribution of type reports of some initial type s_1 consistent with truthful reporting of initial type $r(s_1)$. Note that the expected lifetime labor income of type s_1 from following σ_r^{stat} is equal to the expected lifetime income of initial type $r(s_1)$.

In order to check that the truthful reporting is the equilibrium strategy given (\bar{c}, \bar{y}) , consider any pure reporting strategy $\bar{r} \in \mathcal{R}$. The utility of any initial type from following this strategy given outcomes (\bar{c}, \bar{y}) is equal to the utility from following the strategy $\sigma_{\bar{r}}^{stat}$ with outcomes (c, y) . To see this, note that the utility from consumption and the expected lifetime income generated by $\sigma_{\bar{r}}^{stat}$ given (c, y) are identical to those generated by reporting \bar{r} given (\bar{c}, \bar{y}) . Since lifetime incomes are the same in the two cases, the labor supply allocation will be the same as well. Therefore, the utility from following \bar{r} given (\bar{c}, \bar{y}) is equal to the utility from following $\sigma_{\bar{r}}^{stat}$ given the original mechanism (c, y) . Since in the original mechanism the expected utility from any reporting strategy was bounded above by the truthful reporting, the expected utility from following \bar{r} is also weakly lower than the expected utility from truth-telling in the new mechanism (\bar{c}, \bar{y}) . What remains to be shown is that incentive compatibility holds also for the mixed reporting strategies. For any $\bar{\sigma} \in \Delta_{\mathcal{R}}$ we can define another mixed reporting strategy σ' such that $\sigma'(r') = \sum_{r \in \mathcal{R}} \bar{\sigma}(r) \sigma_r^{stat}(r')$. By the argument above, the expected utility from following $\bar{\sigma}$ given (\bar{c}, \bar{y}) is equal to the expected utility from following σ' given (c, y) . Since the original mechanism implements truth-telling, so does the new mechanism. \square

Proof of Theorem 2.1. First, Lemma 2.8 shows that it is always optimal to implement full consumption insurance allocation, in which the lifetime consumption and the lifetime labor income is determined by the initial type report. It means that we can represent the expected utility of the initial type h_1 that reports type s_1 with $V_{h_1}(C(s_1), Y(s_1))$. Below I show first that at the optimum only the incentive-compatibility constraints with respect to mixed strategies can bind. Secondly,

I prove that (2.9) is a relevant incentive-compatibility constraint with respect to mixed reporting strategies.

First, I will show that only the incentive compatibility constraints corresponding to mixed reporting strategies can be binding. Suppose that there is some pure reporting strategy $r \neq r^*$ such that $V_s(C(s), Y(s)) = V_s(C(r(s)), Y(r(s)))$. Consider reporting strategy σ which mixes between the truthful reporting and r : $\sigma(r^*) + \sigma(r) = 1$, $\sigma(r) \in (0, 1)$. Denote $\bar{Y} \equiv \sigma(r^*)Y(s) + \sigma(r)Y(r(s))$ and take a particular labor allocation \bar{n} defined as $\forall_{s' \in \mathcal{H}(s)} \bar{n}(s') = \sigma(r^*)n_{Y(s)}^{FC}(s') + \sigma(r)n_{Y(r(s))}^{FC}(s')$ which generates \bar{Y} . Since v is strictly convex, $n_{\bar{Y}}^{FC}(s') \neq \bar{n}(s')$, so this labor allocation is suboptimal. Nevertheless, as we will see, the worker prefers to deviate to σ even with a suboptimal labor allocation after deviation. Let's compare the worker's payoff from σ with the payoff from truthful reporting:

$$\begin{aligned}
& \sum_{r' \in \mathcal{R}} \sigma(r') V_s(C(r'(s)), \bar{Y}) - V_s(C(s), Y(s)) \\
&= \sum_{r' \in \mathcal{R}} \sigma(r') (V_s(C(r'(s)), \bar{Y}) - V_s(C(r'(s)), Y(r'(s)))) \\
&= \sum_{s' \in \mathcal{H}(s)} \beta^{|s'|} \mu(s' | s) \left(\sum_{r' \in \mathcal{R}} \sigma(r') v(n_{Y(r(s))}^{FC}(s')) - v(n_{\bar{Y}}^{FC}(s')) \right) \\
&\geq \sum_{s' \in \mathcal{H}(s)} \beta^{|s'|} \mu(s' | s) \left(\sum_{r' \in \mathcal{R}} \sigma(r') v(n_{Y(r(s))}^{FC}(s')) - v(\bar{n}(s')) \right) > 0.
\end{aligned}$$

The first equality comes from the fact that the pure reporting strategy r provides as much utility to initial type s as truthful revelation. Then we can cancel out the utility from consumption, which leads to the second equality. The first inequality is implied by the fact that \bar{n} is not a utility maximizing choice of labor supply that generates \bar{Y} . The final result comes from Jensen's inequality, since v is strictly concave. We have seen that whenever any pure reporting strategy gives as much expected utility as the truthful reporting, the agent would be strictly better off by mixing between the two. Therefore, only the incentive constraints with respect to mixed strategies can bind in the optimum.

Consider the choice of the mixed reporting strategy of some initial type s given the schedules of lifetime consumption C and lifetime income Y :

$$\max_{\sigma \in \Delta_{\mathcal{R}}} \sum_{r \in \mathcal{R}} \sigma(r) V_s \left(C(r(s)), \sum_{r \in \mathcal{R}} \sigma(r) Y(r(s)) \right).$$

The derivative of the objective function with respect to $\sigma(r)$, under normalization $\sigma(r^*) = 1 -$

$\sum_{r \neq r^*} \sigma(r)$, is given by

$$\bar{\beta}u(C(r(s))/\bar{\beta}) - \bar{\beta}u(C(s)/\bar{\beta}) - (Y(r(s)) - Y(s)) \phi_s \left(\sum_{r \in \mathcal{R}} \sigma(r) Y(r(s)) \right),$$

where $\phi_s(Y)$, defined by (2.8), equals $-\partial V_s(C, Y)/\partial Y$. This problem is concave, since $\phi'_s(Y) > 0$. Hence, the necessary and sufficient condition for truth-telling is that the above derivative is non-positive when evaluated at the truthful reporting

$$\forall_{s' \in \Theta_1} \bar{\beta}u(C(s')/\bar{\beta}) - \bar{\beta}u(C(s)/\bar{\beta}) - (Y(s') - Y(s)) \phi_s(Y(s)) \leq 0.$$

The rearrangement of terms generates the incentive-compatibility conditions (2.9). \square

Proof of Proposition 2.1. Take $s, s' \in \Theta_1$ such that $Y(s) > Y(s')$. Then incentive compatibility constraints (2.9) preventing s from mimicking s' and *vice versa* imply that

$$\phi_{s'}(Y(s')) \geq \frac{\bar{\beta} (u(C(s)/\bar{\beta}) - u(C(s')/\bar{\beta}))}{Y(s) - Y(s')} \geq \phi_s(Y(s)).$$

Finally note that $\phi_s(Y(s)) = (1 - T'(Y(s)))u'(C(s)/\bar{\beta})$. \square

Proof of Proposition 2.2. Set up a Lagrangian \mathcal{L} corresponding to the maximization problem in Theorem 2.1. Denote the top type by $\bar{\theta}$. Denote the multiplier w.r.t. the resource constraint by η . We know that in the optimum some downwards incentive constraints of the top type will bind. Moreover, no lower type is tempted to mimic the top type - otherwise, the planner could assign them the top type's income and consumption and get additional resources, since they would end up paying higher taxes. Denote the multipliers with respect to the incentive constraints preventing the top type from mimicking some type θ by ξ_θ . The derivatives of the Lagrangian w.r.t. $C(\bar{\theta})$ and $Y(\bar{\theta})$ are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C(\bar{\theta})} &= \mu(\bar{\theta})u'(C(\bar{\theta})) - \mu(\bar{\theta})\eta + \sum_{\theta \in \Theta_1} \xi_\theta u'(C(\bar{\theta})), \\ \frac{\partial \mathcal{L}}{\partial Y(\bar{\theta})} &= -\mu(\bar{\theta})\phi_{\bar{\theta}}(Y(\bar{\theta})) + \mu(\bar{\theta})\eta - \sum_{\theta \in \Theta_1} \xi_\theta \left(\phi_{\bar{\theta}}(Y(\bar{\theta})) + (Y(\bar{\theta}) - Y(\theta)) \phi'_{\bar{\theta}}(Y(\bar{\theta})) \right). \end{aligned}$$

By setting the derivatives to zero and combining the two equations we get

$$\frac{\phi_{\bar{\theta}}(Y(\bar{\theta}))}{u'(C(\bar{\theta}))} = \frac{\mu(\bar{\theta}) + \sum_{\theta \in \Theta_1} \xi_\theta}{\mu(\bar{\theta}) + \sum_{\theta \in \Theta_1} \xi_\theta \left(1 + (Y(\bar{\theta}) - Y(\theta)) \frac{\phi'_{\bar{\theta}}(Y(\bar{\theta}))}{\phi_{\bar{\theta}}(Y(\bar{\theta}))} \right)}.$$

The term $(Y(\bar{\theta}) - Y(\theta)) \frac{\phi'_\theta(Y(\bar{\theta}))}{\phi_\theta(Y(\bar{\theta}))}$ is positive for all $\theta < \bar{\theta}$, hence the ratio above is smaller than 1. Consequently, the labor supply of the top type is distorted downwards. \square

Proofs from Section 2.4

Proof of Lemma 2.3. Take any (σ, n_σ) which is consistent with limited commitment constraints (2.3) and yields non-negative profits ex ante. Neither workers nor firms can have incentives to terminate the contract (i.e. condition (2.3) is satisfied) for any pure reporting strategy r such that $\sigma(r) > 0$. It is possible only if (σ, n_σ) yields zero profits at each pure reporting strategy that can be drawn with a positive probability. Otherwise there is some pure strategy with a positive probability which yields positive profits, which violates (2.3). Furthermore, $\sum_{r \in \mathcal{R}} \sigma(r) \mathbb{E}U(c \circ r, n_r) \leq \mathbb{E}U(c \circ r', n_{r'})$ for some r' such that $\sigma(r') > 0$. Therefore, $(r', n_{r'})$ yields the same profits and weakly greater utility than (σ, n_σ) . \square

Proof of Lemma 2.4. With fixed-term contract and given a reporting strategy r , the limited commitment constraints mean that $\forall_{h \in \mathcal{H}} \mathbb{E}U_h(c \circ r, n) = 0$. I will show by induction that it implies that income and output coincide at each history. Zero profit conditions at the histories of length \bar{t} imply $\forall_{h \in \mathcal{H}_{\bar{t}}} y(r(h)) = \theta(h) n(h)$. Consider history of length t and suppose that for all histories of length greater than t labor income equals output. Then $\forall_{h \in \mathcal{H}_t} \mathbb{E}\pi_h(c \circ r, n) = \theta(h) n(h) - y(r(h))$, which is equal to zero by the zero profit condition. \square

Proof of Theorem 2.2. Take any allocation under full commitment on the labor market: (c^{FC}, y^{FC}, n^{FC}) , where $(\sigma^*, n^{FC}) \in \mathcal{E}^{FC}(c^{FC}, y^{FC})$. We will find a new allocation of labor income y such that $(\sigma^*, n^{FC}) \in \mathcal{E}(c^{FC}, y, f)$, where only permanent contracts are used: $\forall_{h \in \mathcal{H}} f(h) = \bar{f}$. For any non-initial history $s \in \mathcal{H} \setminus \mathcal{H}_1$ set $y(s) = \max_{s' \in \mathcal{H}_{|s|}(s^{-1})} \theta(s') n(s')$. The limited commitment constraint holds at s : $\mathbb{E}\pi_s(y, n) \leq 0$, as the firm pays the worker at least his output. Then modify the initial labor income such that the zero profit condition holds:

$$\forall_{h \in \mathcal{H}_1} y(h) = y^{FC}(h) - \sum_{s \in \mathcal{H}(h) \setminus \{h\}} R^{1-|s|} \mu(s | h) (y(s) - y^{FC}(s)).$$

As the expected lifetime income of each initial type is unchanged, the zero profit condition holds. Hence, (σ^*, n^{FC}) belongs to the constraint set of the maximization problem that defines $\mathcal{E}(c^{FC}, y, f^P)$. To see that in the constraint set there is no better contract for the worker, note that if there was one, then (σ^*, n^{FC}) would not be an equilibrium contract under full commitment. \square

Lemma 2.9. Suppose that (i) there are two time periods, (ii) there are two productivity levels that are independent over time: $\Theta_1 = \Theta_2 = \{0, 1\}$. Consider allocation (c, y, n) implemented with

mechanism (c, y, f) , where $f(1) = 0$ and c is consistent with the inverse Euler equation. The planner can implement the same allocation with a mechanism (c, y, f') , where $f'(1) = \bar{f}$ and $f'(0) = f(0)$.

Proof of Lemma 2.9. First, note that because of zero productivity the initial low type is unable to mimic the initial high type. Second, I will show that the firm would choose the same labor supply allocation of the initial high type under the new contract assignment. The inverse Euler equation together with fixed term contract implies that $u'(c(1, 1)) > u'(c(1)) > u'(c(1, 0))$. The labor supply $n(1)$ is undistorted since the initial incentive compatibility constraint preventing type 0 from mimicking type 1 is slack. What is more, $n(1, 1)$ is undistorted as well, since insurance against the second period productivity risk requires that type $(1, 1)$ finances consumption of the type $(1, 0)$ ('no distortion at the top'). Hence, we have

$$v'(n(1)) = u'(c(1)) > u'(c(1, 1)) = v'(n(1, 1)).$$

It means that, when the high type has permanent contract, the firm cannot improve his allocation of labor by shifting labor supply from the history $(1, 1)$ to the initial history. Of course, the firm cannot shift labor from the other second period history, since $n(1, 0) = 0$. Therefore, the labor supply allocation of the high type would remain unchanged after the increase in the firing cost. \square

Proof of Theorem 2.3. Consider the mechanism (c, y, f) with the equilibrium (r^*, n) where the top taxpayer $\theta \in \Theta_1$ has a fixed term contract: $f(\theta) = 0$. Consider a new mechanism (c', y', f') in which θ has permanent contract, the full consumption insurance and the same level of utility as before. More insurance means that the planner saves some resources. If any other type $\theta' \in \Theta_1$ wants to mimic θ , assign $(c'(\theta'), y'(\theta'), f'(\theta')) = (c'(\theta), y'(\theta), f'(\theta))$. They are weakly better-off as well. Furthermore, the planner has more resources - the mimicking types become top-taxpayers and pay higher taxes.

\square

Definition 2.8 (Welfare impact decomposition). Consumption allocation c_1 is defined as follows. c_1 is equal to c for all $h \in \mathcal{H} \setminus \mathcal{H}(\underline{\theta})$. For the histories following from the initial type $\underline{\theta}$, it is defined as

$$c_1|_{\mathcal{H}(\underline{\theta})} \in \arg \max_{\tilde{c}: \mathcal{H}(\underline{\theta}) \rightarrow \mathbb{R}_+} \mathbb{E}U_{\underline{\theta}}(\tilde{c}, n)$$

subject to keeping the present value of consumption constant $\sum_{h \in \mathcal{H}(\underline{\theta})} R^{-|h|} \mu(h)(\tilde{c}(h) - c(h)) = 0$ and the incentive constraints corresponding to insurance

$$\begin{aligned} \forall \quad & r \in \mathcal{H}(\underline{\theta}) \rightarrow \mathcal{H}(\underline{\theta}) \quad \mathbb{E}U_{\underline{\theta}}(\tilde{c}, n) \geq \mathbb{E}U_{\underline{\theta}}(\tilde{c} \circ r, \bar{n}(r)). \\ \text{s.t. } & \exists_{r' \in \mathcal{R}} \forall_{h \in \mathcal{H}(\underline{\theta})} r'(h) = r'(h) \end{aligned}$$

Consumption allocation c_2 (and the associated allocation of income y_2) is defined by

$$(c_2, y_2) \in \arg \max_{\substack{\tilde{c} : \mathcal{H} \rightarrow \mathbb{R}_+ \\ \tilde{y} : \mathcal{H} \rightarrow \mathbb{R}}} W(\tilde{c}, n)$$

subject to the budget constraint $\sum_{h \in \mathcal{H}} R^{-|h|} \mu(h)(\tilde{c}(h) - c(h)) = 0$ and the incentive compatibility constraints

$$\forall r \in \mathcal{R} \forall n_r : \mathcal{H} \rightarrow \mathbb{R}_+ \mathbb{E} U_{\underline{\theta}}(\tilde{c}, n) \geq \mathbb{E} U_{\underline{\theta}}(\tilde{c} \circ r, n_r),$$

where n_r satisfy zero profit (2.4) and limited commitment (2.3) constraints given the labor income allocation \tilde{y} . Finally, consumption allocation c' (and the associated allocation of income y') is defined by

$$(c', y') \in \arg \max_{\substack{\tilde{c} : \mathcal{H} \rightarrow \mathbb{R}_+ \\ \tilde{y} : \mathcal{H} \rightarrow \mathbb{R}}} W(\tilde{c}, \tilde{n})$$

subject to the budget constraint $\sum_{h \in \mathcal{H}} R^{-|h|} \mu(h)(\tilde{y}(h) - \tilde{c}(h)) = 0$ and the equilibrium constraint

$$(r^*, \tilde{n}) \in \mathcal{E}(c', y', f').$$

The labor supply allocation n' is the labor allocation corresponding to the truthful reporting strategy in $\mathcal{E}(c', y', f')$.

Proof of Lemma 2.5. $\Delta^{insurance}$ is non-positive, since both c_1 and c have the same net present value, but c_1 has to satisfy additional incentive-compatibility constraints implied by fixed-term contract. Specifically, full consumption insurance of $\underline{\theta}$ is ruled out. $\Delta^{efficiency}$ is non-negative, since the planner can choose outcomes (c_2, y_2, f') . Under these outcomes, the welfare is weakly higher than $W(c_2, n_2)$, as the firms can optimize over labor. Note that when we found the allocation c_2 , the firms were allowed to optimize over labor only when deviating.

Suppose that $\underline{\theta} \in \arg \max_{\theta \in \Theta_1} \lambda(\theta) u'(c_1(\theta))$. It means that the planner wants to redistribute to $\underline{\theta}$. By assigning fixed-term contract to this type, the incentive constraint that prevent redistribution from other initial types to $\underline{\theta}$ are (weakly) relaxed for two reasons: the consumption of $\underline{\theta}$ is more volatile and the fixed-term contract prevents labor smoothing after deviation. Since incentive constraints are relaxed, the planner can redistribute to $\underline{\theta}$, raising the social welfare ($\Delta^{redistribution} \geq 0$). In the second case, when $\underline{\theta} \in \arg \min_{\theta \in \Theta_1} \lambda(\theta) u'(c_1(\theta))$, the planner wants to redistribute from $\underline{\theta}$. However, assigning fixed-term contract weakly decreases utility of this type because of more volatile consumption. Hence, $\underline{\theta}$ is even more tempted to mimic other types and the redistribution has to be reduced, leading to lower welfare ($\Delta^{redistribution} \leq 0$). \square

Definition 2.9. A one-shot deviation $d_{h,h'}$ is a reporting strategy such that (i) $\forall s \in \mathcal{H}/\mathcal{H}(h) d_{h,h'}(s) =$

s and (ii) $\forall_{s \in \mathcal{H}(h)} d_{h,h'}(s) = (h', s_{|h|+1}, \dots, s_{|s|})$.

Lemma 2.10. *Suppose that only fixed-term contracts are used. Under Assumption 2.1, if the incentive constraints with respect to one-shot deviations hold, the incentive constraints with respect to all reporting strategies are satisfied.*

Proof of Lemma 2.10. First I'll prove a useful property that holds under Assumption 2.1. Take any reporting strategy r' and some history h . Construct another reporting strategy r'' such that $r''(s) = s$ for $s \notin \mathcal{H}(r'(h))$ and $r''(s) = r'(h, s_{|h|+1}, \dots, s_{|s|})$ for $s \in \mathcal{H}(r'(h))$. For any $\theta \in \Theta_{|h|+1}$

$$\mathbb{E}U_{(h,\theta)}(c \circ r', \bar{n}(r')) = \mathbb{E}U_{(r'(h),\theta)}(c \circ r'', \bar{n}(r'')). \quad (2.16)$$

The full support assumption guarantees that $(r'(h), \theta)$ is a history with a positive probability. Note that history of reports on both sides of equation are identical. Moreover, the last productivity draw is the same, so by the Markov property productivity distribution is the same. Finally, the function from new productivity draws to reports is identical as well. Hence, the payoffs are equal.

Take any allocation (c, y, n) . I'll show that if (i) the payoff from any reporting strategy r that is truthful before history $h \in \mathcal{H}_{t+1}$ is dominated by the payoff from the one-shot deviation $d_{h,r(h)}$ and (ii) the incentive constraints w.r.t. all one-shot deviations that are truthful before period $t+1$ hold, then the payoff from any reporting strategy r' that is truthful before any history $h' \in \mathcal{H}_t$ is dominated by the payoff from the one-shot deviation $d_{h',r'(h')}$.

Take some reporting strategy r' that is truthful before history $h \in \mathcal{H}_t$ and define r'' as in the first paragraph of this proof. For any $\theta \in \Theta_{t+1}$

$$\begin{aligned} \mathbb{E}U_{(h,\theta)}(c \circ r', \bar{n}(r')) &= \mathbb{E}U_{(r'(h),\theta)}(c \circ r'', \bar{n}(r'')) \\ &\leq \mathbb{E}U_{(r'(h),\theta)}(c \circ d_{(r'(h),\theta),r''(r'(h),\theta)}, \bar{n}(d_{(r'(h),\theta),r''(r'(h),\theta))}) \\ &\leq \mathbb{E}U_{(r'(h),\theta)}(c, n) \\ &= \mathbb{E}U_{(h,\theta)}(c \circ d_{h,r'(h)}, \bar{n}(d_{h,r'(h)})). \end{aligned}$$

The first equality comes from the property (2.16). The second step is implied by the assumption (i) that the one-shot deviations dominate all other reporting strategies that are truthful before period $t+1$. The consecutive inequality is implied by incentive compatibility w.r.t. one-shot deviations. The final equation is again implied by (2.16). Summing up payoffs for all $\theta \in \Theta_{t+1}$, weighted by their conditional probability, and adding the instantaneous utility at the history h leads to

$$\mathbb{E}U_h(c \circ r', \bar{n}(r')) \leq \mathbb{E}U_h(c \circ d_{h,r'(h)}, \bar{n}(d_{h,r'(h)})). \quad (2.17)$$

This inequality means that the payoff from r' is bounded above by the payoff from the corresponding

one-shot deviation, which concludes the proof of the induction step. Finally, note that at the terminal period \bar{t} the only reporting strategies that are truthful before the terminal period are one-shot deviations. Hence, by induction, if incentive constraints with respect to all one-shot deviations are satisfied, the incentive constraints w.r.t. all possible reporting strategies are satisfied. \square

Proof of Proposition 2.3. Suppose that the planner maximizes the utility of $\underline{\theta}$. Denote by $C(\theta)$ and $Y(\theta)$ the present value of consumption and labor income of θ under truthful revelation. Denote by n_0 the labor supply allocation when type $\underline{\theta}$ has permanent contract. When $\underline{\theta}$ has a permanent contract, the incentive constraint that prevents the redistribution from type θ to $\underline{\theta}$ is

$$C(\theta) - \sum_{s \in \mathcal{H}(\theta)} \beta^{|s|-1} \mu(s | \theta) v(n(s)) \geq C(\underline{\theta}) - \sum_{s \in \mathcal{H}(\theta)} \beta^{|s|-1} \mu(s | \theta) v(\tilde{n}(s)), \quad (2.18)$$

where \tilde{n} produces $Y(\underline{\theta})$ and satisfies the labor smoothing condition (2.8) whenever it is consistent with limited commitment. When $\underline{\theta}$ has a fixed-term contract, keeping the net present value of consumption and the allocation of labor unchanged, the analogous incentive constraint is

$$C(\theta) - \sum_{s \in \mathcal{H}(\theta)} \beta^{|s|-1} \mu(s | \theta) v(n(s)) \geq C(\underline{\theta}) - \sum_{s \in \mathcal{H}(\theta)} \beta^{|s|-1} \mu(s | \theta) v(\hat{n}(s)), \quad (2.19)$$

where $\hat{n}(s) = \frac{r_{\theta, \underline{\theta}}(s)}{\theta(s)} n_0(s)$. First, by Lemma 2.10 we can focus only on one-shot deviation. Second, since future productivities are independent of initial draw, the net present value of consumption of deviating type θ is equal to $C(\underline{\theta})$ and $Y(\underline{\theta})$. I will show that the right-hand side of (2.18) is strictly greater than of (2.19). Note that \tilde{n} and \hat{n} produce the same output $Y(\underline{\theta})$. Moreover, the labor supply allocation n satisfies

$$\forall_{s \in \mathcal{H}(\underline{\theta})} \frac{v'(n(\underline{\theta}))}{\underline{\theta}} = \frac{v'(n(s))}{\theta(s)}.$$

Since $\theta > \underline{\theta}$, this implies that

$$\forall_{s \in \mathcal{H}(\theta)} \frac{v'(\hat{n}(\theta))}{\theta} < \frac{v'(\hat{n}(s))}{\theta(s)}.$$

The initial type θ could reduce the disutility from labor by producing more in the initial period, which does not violate the limited commitment constraints. It means that $\hat{n}(\theta) < \tilde{n}(\theta)$ and hence the right-hand side of (2.18) is strictly greater than of (2.19). Hence, the incentive constraint is relaxed when $\underline{\theta}$ receives fixed-term contract. Since it is true for all types $\theta \in \Theta \setminus \underline{\theta}$, we have $\Delta^{\text{redistribution}} > 0$. \square

Proof of Proposition 2.4. Note that assigning fixed-term contract to $\underline{\theta}$, while keeping outcome functions c and y constant, does not change the utility from truthfully or untruthfully reporting $\underline{\theta}$ ($\Delta^{\text{insurance}} = \Delta^{\text{redistribution}} = 0$). Below I show that when $\underline{\theta}$ has fixed-term contract, the planner

can perturb the outcome functions of this type to save resources without changing his utility level nor violating any incentive constraint.

Take history $h \in \mathcal{H}(\underline{\theta})$ with $\theta(h) = \max \Theta_{|h|}$ at which the labor supply is distorted downwards: $\theta(h)u'(c(h)) > v'(y(s)/\theta(s))$. Perturb consumption and income such that $\theta(h)u'(c'(h)) = v'(y'(h)/\theta(h))$ and the instantaneous utility at this history is unchanged. Since distortions are lifted, the planner obtains additional resources. Furthermore, since h is the most productive type, lifting the downward distortion relaxes the incentive constraints w.r.p. other types $h' \in \mathcal{H}(h^{-1})$. The incentive constraints corresponding to deviations at earlier dates are unaffected by Assumption 2.1 and Lemma 2.10. The utility from a one shot deviation $\mathbb{E}U_{(s,\theta)}(c \circ d_{s,h^{-1}}, n_{d_{s,h^{-1}}}) = \mathbb{E}U_h(c, n)$ is unaffected by the perturbation for any $\theta \in \Theta_{|h|}$. Hence, when $\underline{\theta}$ has fixed-term contract, the planner can lift some future distortions and obtain additional resources without violating incentive compatibility. These resources can be spend on uniform raise of expected utility of all types, leading to $\Delta^{efficiency} > 0$. \square

Proofs from Section 2.5

Lemma 2.11. *Under Assumption 2.2 the function $V_\theta(C, Y)$ satisfies the Spence-Mirrlees single-crossing condition.*

Proof of Lemma 2.11. The Spence-Mirrlees condition states that $-\frac{\partial V_\theta(C, Y)}{\partial Y} \left(\frac{\partial V_\theta(C, Y)}{\partial C} \right)^{-1}$ is non-increasing with θ . Since the denominator is positive and constant in θ , it is enough to show that $\frac{\partial V_\theta(C, Y)}{\partial Y}$ is non-decreasing with θ .

Note that $\frac{\partial V_\theta(C, Y)}{\partial Y} = -\frac{v'(n(h))}{\theta(h)}$ for any $h \in \mathcal{H}(\theta)$. If we fix Y , the labor supply depends only on the initial type h and current productivity θ , so we can write it as $n_h(\theta)$. Note that $n_h(\theta)$ is increasing in θ . Let's define the extension of $n_{h_1}(\theta)$ to all possible productivity realizations with a step function $\bar{n}_h(\theta) \equiv \max_{\theta' \in \Theta(h) \cap [0, \theta]} n_h(\theta')$, where $\Theta(h) \equiv \{\theta(s) : s \in \mathcal{H}(h)\}$ is a set of all possible productivity realizations following the history h .

Suppose that $\frac{\partial V_{h_1}(C, Y)}{\partial Y} > \frac{\partial V_{s_1}(C, Y)}{\partial Y}$ for some initial types $s_1 > h_1$. It means that for any productivity level $\theta \in \Theta(h_1) \cup \Theta(s_1)$ we have $n_{s_1}(\theta) > n_{h_1}(\theta)$. Now we have

$$\sum_{\theta \in \mathbb{R}_+} \hat{\mu}(\theta | h_1) \theta n_{h_1}(\theta) = \sum_{\theta \in \mathbb{R}_+} \hat{\mu}(\theta | h_1) \theta \bar{n}_{h_1}(\theta) \leq_{FOSD} \sum_{\theta \in \mathbb{R}_+} \hat{\mu}(\theta | s_1) \theta \bar{n}_{h_1}(\theta) < \sum_{\theta \in \mathbb{R}_+} \hat{\mu}(\theta | s_1) \theta n_{s_1}(\theta).$$

The weak inequality is implied by the first-order stochastic dominance (Assumption 2.2), since $\bar{n}_{h_1}(\theta)$ is a non-decreasing function of θ . The second inequality comes from $n_{s_1}(\theta) > n_{h_1}(\theta') = \bar{n}_{h_1}(\theta)$ for some $\theta' \leq \theta$. The left-hand side is the lifetime income of initial type h_1 divided by $\sum_{t=1}^{\bar{t}}$, while the right-hand side is the lifetime income of s_1 . Since we assumed that their lifetime incomes are both equal to Y , we have a contradiction. Therefore $\frac{\partial V_\theta(C, Y)}{\partial Y}$ is non-decreasing in θ . \square

Proof of Proposition 2.5 . The solution to the Mirrlees model, when the first order approach is valid, was expressed in terms of elasticities by Saez (2001). The first-order approach is valid if the single crossing condition holds and the resulting income schedule is non-decreasing. The single crossing holds by Assumption 2.2 and Lemma 2.11. Hence, what remains to be shown are the relevant elasticities, which I derive below.

First, recall the definition of $\phi_\theta(Y)$ in (2.8). Let's define the marginal tax rate $T'(Y(\theta))$ as $1 - \frac{\phi_\theta(Y(\theta))}{u'(C(\theta)/\bar{\beta})}$. I will derive the elasticities by varying the marginal tax rate. The compensated elasticity is given by

$$\bar{\zeta}^c = \frac{\partial Y(\theta)}{\partial 1 - T'(\theta)} \Big|_{dC(\theta)=0} \frac{1 - T'(\theta)}{Y(\theta)} = \frac{\phi(\theta, Y(\theta))}{\phi'_\theta(Y(\theta)) Y(\theta)}.$$

We can derive ϕ' with the implicit function theorem. First, we can use (2.8) express $n(h)$ as $g(\theta(h) \phi_{h_1}(Y(h_1)))$, where g is an inverse function of v' . Plug this expression into the zero profit condition to get

$$H = \sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta) \theta(h) g(\theta(h) \phi_\theta(Y(\theta))) - Y(\theta) = 0.$$

By the implicit function theorem we have

$$\begin{aligned} \phi'_\theta(Y(\theta)) &= -\frac{\partial H}{\partial Y(\theta)} \left(\frac{\partial H}{\partial \phi_\theta(Y(\theta))} \right)^{-1} = \left(\sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta) (\theta(h))^2 g'(\theta(h) \phi_\theta(Y(\theta))) \right)^{-1} = \\ &= \left(\sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta) \frac{(\theta(h))^2}{v''(n(h))} \right)^{-1}. \end{aligned}$$

Hence, we have

$$\bar{\zeta}^c = Y(\theta)^{-1} \sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta_1) \frac{\theta(h) n(h) v'(n(h))}{n(h) v''(n(h))} = \sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta_1) \frac{\theta(h) n(h)}{Y(\theta)} \zeta^c(h),$$

where $\zeta^c(h)$ is the compensated elasticity of labor supply at history h . The lifetime compensated elasticity is the average compensated elasticity across all histories starting in θ , weighted by the realized output. The uncompensated elasticity is given by

$$\bar{\zeta}^u = \frac{\partial Y(\theta)}{\partial 1 - T'(\theta)} \frac{1 - T'(\theta)}{Y(\theta)} = \bar{\zeta}^c + \frac{u''(C(\theta)/\bar{\beta})}{\beta \phi'_\theta(Y(\theta))} (1 - T'(\theta)).$$

Denote the wealth effect by $\bar{\xi} = \bar{\zeta}^c - \bar{\zeta}^u$. Then

$$\bar{\xi} = \bar{\beta}^{-1} \left(\sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta) \frac{(\theta(h))^2 (1 - T'(\theta)) u''}{v''(n(h))} \right) = \bar{\beta}^{-1} \sum_{h \in \mathcal{H}(\theta)} R^{1-|h|} \mu(h | \theta) \xi(h),$$

where $\xi(h)$ is the wealth effect at the history h . The lifetime wealth effect is the average wealth effect across all histories. Now what remains to be done is to plug the derived elasticities in the [Saez \(2001\)](#) formula. \square

Proof of Lemma 2.6. It follows from Lemma 2.2, with individual consumption determined by the usual budget constraint. \square

Proof of Lemma 2.7. We can apply the proof of Theorem 2.2. \square

Proof of Theorem 2.4. First I will show that there exists a history dependent asset policy a such that, given the tax T , the allocation together with a belongs to $\tilde{\mathcal{E}}(T)$. Define $a(h) = \sum_{t=1}^{|h|} R^{1-|h|} (y(h^t) - \bar{y}(h_1))$. Then at each history consumption expenditure is equal $\bar{y}(h_1)$. Hence at each history $T_x(x(h)) = \bar{T}(\theta)$ and the right-hand side of the budget constraint equals $c(h)$.

Suppose that \bar{T}_x is convex. Take some initial type θ . He may deviate either to a constant, but different level of consumption expenditures, or he may introduce some volatility to consumption expenditures. The first deviation is taken care of by the equilibrium constraint from the corresponding direct mechanism as well as a punitively high tax whenever worker deviates to a level that does not correspond to consumption expenditures of any other tax. The introduction of volatility in consumption expenditures with the expected value \bar{x} means, due to convexity of the tax system, that the expected tax paid is not lower than $T_x(\bar{x})$. Since the labor supply allocation is the same both cases (labor allocation depends on expected lifetime income, which is equal to the expected lifetime expenditure), the utility from introducing volatility is bounded above by a utility of deviation to the constant consumption expenditure \bar{x} (which was taken care of above). Note that deviations to fixed-term contract imply volatile consumption expenditures and are not tempting by the same argument.

When \bar{T}_x is not convex, we can add an auxiliary correction term $\alpha(x_t - x_0)^2$ which punishes volatile consumption expenditures. The parameter α should be high enough such $\bar{T}_x(x_t) + \alpha(x_t - x_0)^2$ is convex. Then the reasoning above applies. \square

Proof of Proposition 2.6. I will show that the inverse Euler equation holds. The proof follows [Goloso, Kocherlakota, and Tsyvinski \(2003\)](#). Take any allocation (c, y, n) implemented by some

direct mechanism (c, y, f) . Take some history $h \in \mathcal{H} \setminus \mathcal{H}_{\bar{t}}$ and consider a small perturbation δ such that

$$c'(h) = c(h) + \frac{\delta}{u'(c(h))}, \quad \forall s \in \mathcal{H}_{|h|+1}(h) \quad c'(s) = c(s) - \frac{\delta}{\beta u'(c(s))},$$

and c' equal c elsewhere. As the utility from any reporting strategy is unchanged, truthful revelation still holds in equilibrium. In the optimum such perturbation cannot yield free resources

$$-\frac{\delta \mu(h)}{u'(c(h))} + \sum_{s \in \mathcal{H}(h)_{|h|+1}} \frac{\mu(s|h) \delta}{\beta u'(c(s))} = 0,$$

which implies that the inverse Euler equation holds. To see how the inverse Euler equation together with volatile consumption implies a savings distortion and capital tax, see for instance [Goloso](#), [Kocherlakota](#), and [Tsyvinski](#) (2003). \square

Auxiliary estimates

Table 2..1: The regression of $\log\left(\frac{y_{ijt}}{y_{ijt-1}}\right)$ for permanent contracts.

variable	coef	std err	t	p-value
const	0.0285***	0.001	27.766	0.000
male	0.0072***	0.001	11.940	0.000
tenure	-0.0015***	4.97e-05	-30.120	0.000
d1999	-0.0094***	0.001	-11.243	0.000
d2000	-0.0073***	0.001	-8.857	0.000
d2001	-0.0056***	0.001	-6.812	0.000
d2002	-0.0095***	0.001	-11.697	0.000
white_colar	0.0169***	0.001	28.766	0.000
cadre	0.0106***	0.001	7.288	0.000
manager	-0.0391***	0.002	-18.043	0.000
North-East	0.0008	0.001	1.274	0.202
Center	-0.0036***	0.001	-4.959	0.000
South	-0.0056***	0.001	-6.717	0.000
Islands	-0.0055***	0.001	-4.616	0.000
young_firm	0.0011*	0.001	1.660	0.097
agriculture	-0.0096**	0.005	-2.093	0.036
heavy_industry	-0.0072***	0.002	-3.892	0.000
manufacturing	-0.0038***	0.001	-6.149	0.000
construction	-0.0174***	0.001	-16.560	0.000
services1	0.0002	0.001	0.224	0.823

Table 2..2: The regression of $\log\left(\frac{y_{ijt}}{y_{ijt-1}}\right)$ for fixed-term contracts.

variable	coef	std err	t	p-value
const	0.0388***	0.010	3.753	0.000
male	0.0031	0.006	0.541	0.588
tenure	-0.0037***	0.001	-3.464	0.001
d1999	-0.0042	0.010	-0.421	0.674
d2000	0.0053	0.009	0.558	0.577
d2001	-6.927e-05	0.009	-0.007	0.994
d2002	-5.987e-05	0.009	-0.007	0.994
white_colar	0.0323***	0.006	5.358	0.000
cadre	0.0103	0.034	0.302	0.763
manager	-0.0192	0.037	-0.517	0.605
North-East	-0.0111*	0.007	-1.643	0.100
Center	-0.0165**	0.007	-2.293	0.022
South	-0.0031	0.009	-0.341	0.733
Islands	-0.0213*	0.011	-1.902	0.057
young_firm	-0.0047	0.007	-0.698	0.485
agriculture	-0.0392	0.041	-0.955	0.339
heavy_industry	-0.0006	0.051	-0.012	0.991
manufacturing	-0.0196***	0.006	-3.096	0.002
construction	-0.0313**	0.013	-2.370	0.018
services1	0.0085	0.011	0.765	0.445

Table 2..3: The regression of $\hat{\varepsilon}_t^2$ - full results.

variable	coef	std err	t	p-value
const	0.0347***	0.003	10.557	0.000
C_fixed_term	0.0090***	0.001	13.058	0.000
log_income	-0.0019***	0.000	-5.591	0.000
male	-0.0041***	0.000	-17.433	0.000
tenure	-0.0002***	1.91e-05	-12.155	0.000
d1999	0.0005*	0.000	1.747	0.081
d2000	0.0005*	0.000	1.644	0.100
d2001	-0.0003	0.000	-0.996	0.319
d2002	-0.0007**	0.000	-2.257	0.024
white_colar	0.0013***	0.000	5.330	0.000
cadre	0.0025***	0.001	4.130	0.000
manager	-0.0047***	0.001	-5.317	0.000
North-East	-0.0013***	0.000	-5.223	0.000
Center	0.0002	0.000	0.916	0.360
South	0.0018***	0.000	6.069	0.000
Islands	-0.0003	0.000	-0.794	0.427
young_firm	0.0017***	0.000	7.015	0.000
agriculture	-0.0066***	0.002	-3.982	0.000
heavy_industry	-0.0017	0.001	-2.505	0.012
manufacturing	-0.0002	0.000	-0.837	0.402
construction	0.0023***	0.000	6.066	0.000
services1	0.0011***	0.000	3.460	0.001

Table 2..4: The regression of $\hat{\varepsilon}_{t-1}\hat{\varepsilon}_t$ - full results.

variable	coef	std err	t	p-value
const	0.0031	0.002	1.383	0.167
C_fixed_term	-0.0012	0.001	-1.568	0.117
log_income	-0.0006***	0.000	-2.742	0.006
male	0.0018***	0.000	11.332	0.000
tenure	-2.907e-05**	1.3e-05	-2.243	0.025
white_colar	0.0008***	0.000	5.051	0.000
cadre	0.0007*	0.000	1.746	0.081
manager	0.0031***	0.001	5.273	0.000
North-East	0.0003**	0.000	1.999	0.046
Center	-0.0004**	0.000	-1.987	0.047
South	-0.0010***	0.000	-5.057	0.000
Islands	-0.0003	0.000	-0.895	0.371
young_firm	-0.0002	0.000	-1.172	0.241

3 Minimal Compensation and Incentives for Effort

Abstract

When does paying a strictly positive compensation in every state of the world improves incentives to exert effort? I show that in the typical model of moral hazard it happens only when the effort is a strict complement to consumption. If the cost of effort is monetary, a positive minimal compensation strengthens incentives only when the agent is prudent and always does so when the marginal utility of consumption is unbounded at zero consumption. I discuss potential applications of these results in personal income taxation.

3.1 Introduction

The model of moral hazard demonstrates of the trade-off between insurance and incentives. A risk neutral principal wants to motivate a risk averse agent to exert effort. Moreover, the principal needs to provide the agent with some minimal level of utility, e.g. due to the agent's participation decision. The trade-off exists, since the efficient provision of utility requires the full insurance of the agent, which undermines any incentives for effort. In this paper I show that this trade-off is not absolute: sometimes increasing insurance benefits incentives. I identify cases in which the optimal compensation of the agent includes a positive unconditional pay, even though the principal is not obliged to provide the agent with any minimal level of utility. Thus, the unconditional pay plays a role of the incentive pay, as it strengthens the agent's willingness to exert effort.

In my framework the agent chooses whether to exert effort or not, which affects the distribution of output. The principal, who observes only the realized output, sets up a compensation scheme to motivate the agent to exert effort at the lowest cost. The principal is constrained only by incentive compatibility - the agent needs to be better off by exerting effort. I impose no participation or individual rationality constraints. The sole role of the compensation scheme is to provide incentives for effort.

I am grateful for valuable comments of Árpád Ábrahám and Ramon Marimon. All mistakes are mine.

I study when the optimal compensation scheme includes a positive minimal compensation regardless of the realized output.¹ First, a positive minimal pay is optimal only if effort is a complement to consumption. Only then higher consumption reduces the cost of effort and relaxes the incentive compatibility constraint. Consecutively, I focus on the classical case of complementarity between consumption and effort - the model with a monetary cost of effort. When the output distribution is sufficiently rich,² the agent will be compensated in every state of the world only if he is prudent, i.e. only when the marginal utility of consumption is convex. Without prudence, paying the agent in all the states that are more likely without effort undermines incentives. Finally, a sufficient condition for a positive minimal pay for an arbitrary distribution of output is an unbounded marginal utility of consumption at 0. This simple condition means that marginally increasing the agent compensation above zero always raises the expected utility from exerting effort more strongly than the expected utility from shirking.

[Grossman and Hart \(1983\)](#) study various features of the optimal compensation scheme in the moral hazard problem, such as monotonicity and concavity with respect to output realization. My paper is concerned with the particular feature: the minimal compensation level. [Mirrlees \(1999\)](#) provide conditions under which the first-best outcome to be approximated with a step function with two compensation level. As the lower compensation level converges to zero, the agent's increased effort makes realization of the low pay unlikely. I characterize the polar case, in which the minimal payment to the agent is optimally bounded away from zero. [Holmstrom and Milgrom \(1991\)](#) propose another environment, based on multitasking, in which insurance is good for incentives. Compensation which depends on observed outcomes makes the agent shift the effort away from tasks with unobserved outcomes. As a result, the optimal contract may specify a fix wage which does not depend on the observed outcomes. In my paper, I show that a certain amount of insurance can improve incentives in the standard model with a single task.

I discuss the application of my results in the design of the optimal tax systems. Effort can be interpreted either as an investment in a risky venture or a costly education decision which affects future distribution of income. When the marginal utility of consumption is unbounded at zero, taxing the high income agents and providing positive transfer to the low income agents actually improves incentives for effort. The minimal compensation can be understood as a basic income - an unconditional cash transfer to any agent. [Van Parijs \(1991\)](#) justifies the basic income on the moral grounds. I provide conditions under which the basic income has a positive impact on incentives and can be justified on the efficiency grounds.

¹[Hölmstrom \(1979\)](#) shows that the optimal compensation should vary with any observable variable that is informative of the agents' level of effort. The unconditional pay does not violate this informativeness principle as long as there is a state-contingent bonus on top of it.

²There are at least two output levels which are more likely without effort.

Structure of the paper. The next section introduces the framework. Section 3.3 presents the main theoretical results. They are illustrated by the numerical exercise in Section 3.4. The consecutive section proposes the application of the theory in taxation. The last section concludes and discusses possible extensions.

3.2 Model

The agent chooses whether to exert effort ($e = 1$) or not ($e = 0$).³ The effort affects the distribution of output, which has a finite support $Y \subset \mathbb{R}_+$ and the probability mass function $p_e : Y \rightarrow [0, 1]$. The agent's flow utility function $U(c, e) : \mathbb{R}_+ \times \{0, 1\} \rightarrow \mathbb{R}$ is increasing, strictly concave and twice differentiable in consumption c . The effort is costly: $U(c, 0) - U(c, 1) > 0$ for all $c > 0$, and I assume that this difference is strictly positive in the limit as $c \rightarrow 0$.⁴

The principal does not observe the effort and compensates the agent with payments $w : Y \rightarrow \mathbb{R}_+$ which depend only on the realized output. The optimal compensation scheme solves

$$\max_{w: Y \rightarrow \mathbb{R}_+} \sum_{y \in Y} p_1(y)(y - w(y))$$

subject to the incentive compatibility constraint, guaranteeing that the agent is better off by exert effort

$$\sum_{y \in Y} p_1(y)U(w(y), 1) \geq \sum_{y \in Y} p_0(y)U(w(y), 0). \quad (IC)$$

Note that the agent does not make a participation decision, nor is the principal committed to provide the agent with any minimal level of utility. The only role of the compensation scheme w is to provide agent with incentives for effort. I assume that there exists a compensation scheme which implements a positive effort.⁵ Under this assumption, the principal always prefers to motivate effort if the difference in expected output with and without effort is sufficiently high. I assume that this is the case.

³I focus on the binary effort decision, which simplifies the analysis as I need to consider only one incentive-compatibility constraint. I discuss the extension to the case of continuous effort in the last section.

⁴Otherwise, the agent that receives no compensation at all is indifferent between the two levels of effort and the optimal compensation is trivially equal to 0.

⁵This assumption may be wrong if the effort cost is sufficiently high. Suppose that $Y = \{\underline{y}, \bar{y}\}$, $p_1(\bar{y}) = p$, $p_1(\underline{y}) = 1 - p$, $p_0(\bar{y}) = 0$, $p_0(\underline{y}) = 1$. With the utility function $U(c, e) = -e^{-\gamma(c + (1-e)\epsilon)}$ the incentive compatibility constraint (IC) can be satisfied only if $\gamma\epsilon < -\log(1 - p)$.

3.3 When is the minimal compensation strictly positive?

The three propositions below characterize conditions under which a positive minimal compensation is optimal.

Proposition 3.1. *The minimal compensation is zero if (i) there exists an outcome realization that is possible only when effort is absent or (ii) effort is a substitute to consumption: $\forall c > 0 U_c(c, 0) \geq U_c(c, 1)$.*

Proof. It is optimal to pay the agent only if it relaxes the incentive compatibility constraint. Suppose that there is $\bar{y} \in Y$ which is possible only without effort: $p_1(\bar{y}) = 0$ and $p_0(\bar{y}) > 0$. Increasing $w(\bar{y})$ always tightens (IC), so optimally $w(\bar{y}) = 0$.

Denote the marginal utility from consumption by $U_c(c, e)$. The optimal contract involves positive compensation $w(y) > 0$ for some output $y \in Y$ only if

$$U_c(w(y), 1) - \frac{p_0(y)}{p_1(y)} U_c(w(y), 0) = \frac{1}{\mu} \quad (3.1)$$

where $\mu > 0$ is the Lagrange multiplier of the incentive constraint. On the other hand, $w(y) = 0$ can be optimal only if

$$U_c(0, 1) - \frac{p_0(y)}{p_1(y)} U_c(0, 0) \leq \frac{1}{\mu}. \quad (3.2)$$

If $\forall c > 0 U_c(c, 0) \geq U_c(c, 1)$ then the left-hand side of (3.1) is negative for any output realization that is more likely without effort. Hence, if $p_0(y) > p_1(y)$, which is true for at least one $y \in Y$, then optimally $w(y) = 0$. \square

The agent should not be compensated for output which unambiguously identifies the missing effort. This result is closely related to the ‘unpleasant theorem’ of [Mirrlees \(1999\)](#), according to which the principal can motivate the agent by introducing severe punishments for output levels that are unlikely under the positive effort. Furthermore, the principal will not use the positive minimal pay when consumption is a substitute to effort. When consumption and effort are substitutes, the utility cost of effort $U(c, 0) - U(c, 1)$ weakly increases with consumption. In order to keep the expected cost of effort low, the principal will pay the agent only for output levels which coincide with the positive effort. Specifically, with the commonly assumed additively separable disutility of effort the optimal contract always involve zero minimal pay.

Proposition 3.1 shows that the complementarity between consumption and effort is required for the positive minimal pay. In the remaining part of the paper I derive a sharper characterization of the minimal pay under the classical case of such complementarity - a situation in which the effort has a purely monetary cost.

Assumption 3.1. *The utility function is $U(c, e) = u(c + (1 - e)\epsilon)$, where $u \in \mathcal{C}^3$ is increasing and strictly concave and $\epsilon > 0$. Moreover, no output realization unambiguously identifies the missing effort: $p_0(y) > 0 \implies p_1(y) > 0$.*

Under Assumption 3.1 the effort has a fixed monetary cost $\epsilon > 0$. Hence, by shirking and not incurring the cost, the agent can increase his consumption.

Proposition 3.2. *Under Assumption 3.1, the minimal compensation is zero if any of the following conditions hold:*

1. *The utility function satisfies $u'''(c) \leq 0$ for all $c > 0$ and there are at least two output levels that are more or equally likely without effort.*
2. *The utility function u has a constant absolute risk aversion.*

Proof. [1.] First I will derive an additional necessary optimality condition. Take two output realizations $y, y' \in Y$ such that both $w(y)$ and $w(y')$ satisfy (3.1) and at least one of them is positive. Perturb $w(y)$ by a small δ and $w(y')$ by $-\frac{p_1(y)}{p_1(y')}\delta$. This perturbation does not affect the principal's profit if the effort is unchanged. Define $V_{\bar{y}}(w) \equiv u(w) - \frac{p_0(\bar{y})}{p_1(\bar{y})}u(w + \epsilon)$. The impact of the perturbation on (IC), taking into consideration the terms up to the second order, is

$$\delta \left(p_1(y)V_y'(w(y)) - \frac{p_1(y)}{p_1(y')}p_1(y')V_{y'}'(w(y')) \right) + \frac{\delta^2}{2} \left(p_1(y)V_y''(w(y)) + \left(\frac{p_1(y)}{p_1(y')} \right)^2 p_1(y')V_{y'}''(w(y')) \right).$$

Optimality requires that this expression is non-positive, since otherwise it would be possible to relax the incentive-compatibility constraint without losses in profits. The first-order component is zero by the necessary condition (3.1). Hence, the optimal contract satisfies

$$p_1(y')V_y''(w(y)) + p_1(y)V_{y'}''(w(y')) \leq 0. \quad (3.3)$$

Now, take any $\bar{y} \in Y$ such that $\frac{p_0(\bar{y})}{p_1(\bar{y})} \geq 1$. When $u''' \leq 0$, we have $1 \geq \frac{u''(w(\bar{y}))}{u''(w(\bar{y}) + \epsilon)}$, which together implies that $V_{\bar{y}}''(w(\bar{y})) \geq 0$. If there are two such output levels, then they violate the necessary optimality condition (3.3) unless compensation in both states is 0 or (3.1) holds for at most one of them. Either way, for at least one of these output levels the compensation is optimally 0.

[2.] Suppose that the utility is CARA ($u(c) \equiv -e^{-\gamma c}$) and that the incentive constraint holds as equality for some compensation scheme w with a positive minimal pay \underline{w} . Note that

$$\sum_{y \in Y} p_1(y)e^{-\gamma w(y)} = \sum_{y \in Y} p_0(y)e^{-\gamma(w(y) + \epsilon)} \implies \sum_{y \in Y} p_1(y)e^{-\gamma(w(y) - \underline{w})} = \sum_{y \in Y} p_0(y)e^{-\gamma(w(y) - \underline{w} + \epsilon)},$$

so the principal can save resources by uniformly reducing the compensation in every contingency. \square

When the output distribution is sufficiently rich - there are at least two output levels that are less likely with the positive effort - prudence ($u''' > 0$) becomes the necessary condition for the positive minimal pay. Without prudence, any contract that satisfies (3.1) at each output level violates the second order condition for the local maximum. The principal's problem is convex in compensation for the output levels which are less likely under effort. Then it is optimal to keep the compensation positive for at most one of these output levels. [Rothschild and Stiglitz \(1971\)](#) show that prudent individuals save more when faced with more risk. The effort decision resembles the savings decision, as effort, besides changing the distribution of output, reduces the agents consumption by a constant amount in each contingency. Prudent agents, when faced with less income risk because of the higher minimal compensation, are willing to save less, i.e. exert more effort. This analogy, however, has its limits. The precautionary saving motive increases in the absolute prudence $-u'''/u''$, as demonstrated by [Kimball \(1990\)](#). Nevertheless, even an arbitrarily high level of the absolute prudence is not sufficient to guarantee the optimum with a positive minimal compensation. The CARA utility function, for which the absolute prudence equals the absolute risk aversion, always involves the lowest compensation of zero. In this case, since the agent preferences over lotteries are independent of wealth, providing an unconditional income does not affect the incentive constraint.

Proposition 3.3. *Suppose that Assumption 3.1 holds. The minimal compensation is positive **if** $\lim_{c \rightarrow 0} u'(c) = +\infty$. When the utility function has non-increasing absolute risk aversion, the minimal compensation is positive for an arbitrary distribution of output **only if** $\lim_{c \rightarrow 0} u'(c) = +\infty$.*

Proof. The necessary condition for the corner solution (3.2) is never satisfied when $\lim_{c \rightarrow 0} u'(c) = +\infty$, which proves the 'if' part. To prove the 'only if' part, suppose that $u'(0)$ is finite and construct a distribution of output with some $\bar{y} \in Y$ such that $\frac{p_0(\bar{y})}{p_1(\bar{y})} = \frac{u'(0)}{u'(\epsilon)}$. When the absolute risk aversion is non-increasing, $\frac{u'(c)}{u'(c+\epsilon)}$ is non-increasing with c , since $\frac{\partial}{\partial c} \frac{u'(c)}{u'(c+\epsilon)} = (a(c+\epsilon) - a(c)) \frac{u'(c)}{u'(c+\epsilon)}$, where $a(c) \equiv -\frac{u''(c)}{u'(c)}$ stands for the absolute risk aversion. Hence, for any $w(\bar{y}) \geq 0$ the left-hand side of (3.1) is non-positive and the only possible solution lays in the corner with $w(\bar{y}) = 0$. \square

An unbounded marginal utility from consumption at zero is a sufficient condition for a positive minimal pay. The shirking agent has higher consumption, since he does not incur the cost ϵ . When $\lim_{c \rightarrow 0} u'(c) = +\infty$ and compensation is zero at some output level, the differences in marginal utilities with and without effort dominate any possible difference in odds. As a result, a marginal increase in compensation for any output level, starting from 0, improves the agent's expected utility from exerting effort in comparison to shirking. In other words, the principal can decrease the utility cost of exerting effort $u(c+\epsilon) - u(c)$ by a large amount by marginally increasing the minimal compensation above zero. This result is apparent when $\lim_{c \rightarrow 0} u(c) = -\infty$: without the positive minimal compensation the expected utility of exerting effort is $-\infty$, while the expected

utility of shirking is finite. However, the results holds also for utility functions taking finite values at zero consumption, e.g. CRRA utility with the relative risk aversion lower than 1.

Under plausible conditions, the unbounded marginal utility at zero becomes a necessary condition for a positive minimal pay for an arbitrary distribution of output. For this result to hold, we need a non-increasing absolute risk aversion. Individual preferences satisfy this realistic property if and only if the propensity to take risks does not decrease with wealth. If, on the contrary, the absolute risk aversion is increasing and the marginal utility of consumption is bounded, then we can always find a distribution of output which would imply a zero minimal compensation. Finally, note that Propositions 3.2 and 3.3 are consistent with each other, since an unbounded marginal utility at zero implies prudence in the neighborhood of zero consumption.⁶

3.4 Numerical example

Assumption 3.2. *The utility function is $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, $\sigma > 0$. There are two possible output realizations $Y = \{0, \bar{y}\}$. The high output realization is possible only under effort: $p_1(\bar{y}) = p \in (0, 1)$, $p_0(\bar{y}) = 0$.*

Lemma 3.1. *Under Assumption 3.2 the optimal contract is linear in the cost of effort, i.e. for any σ and p there exist $\omega(\sigma, p)$ and $\beta(\sigma, p)$ such that the optimal contract satisfies*

$$w(0) = \omega(\sigma, p)\epsilon, \quad w(\bar{y}) = w(0) + \beta(\sigma, p)\epsilon. \quad (3.4)$$

Proof. In the Appendix. □

Under simplifying Assumption 3.2 the optimal contract is linear in effort cost ϵ . The compensation can be described with two coefficients: ω , which stands for the guaranteed pay, and β , which stands for the bonus for the high output realization. I compare the optimal contract with the ‘no insurance’ contract, in which the agent is paid only when the high output is realized. In such contract the incentive provision requires that the agents receives $\beta'(\sigma, p)\epsilon$ in the high output state, where $\beta'(\sigma, p) \equiv p^{\frac{1}{\sigma-1}}$.

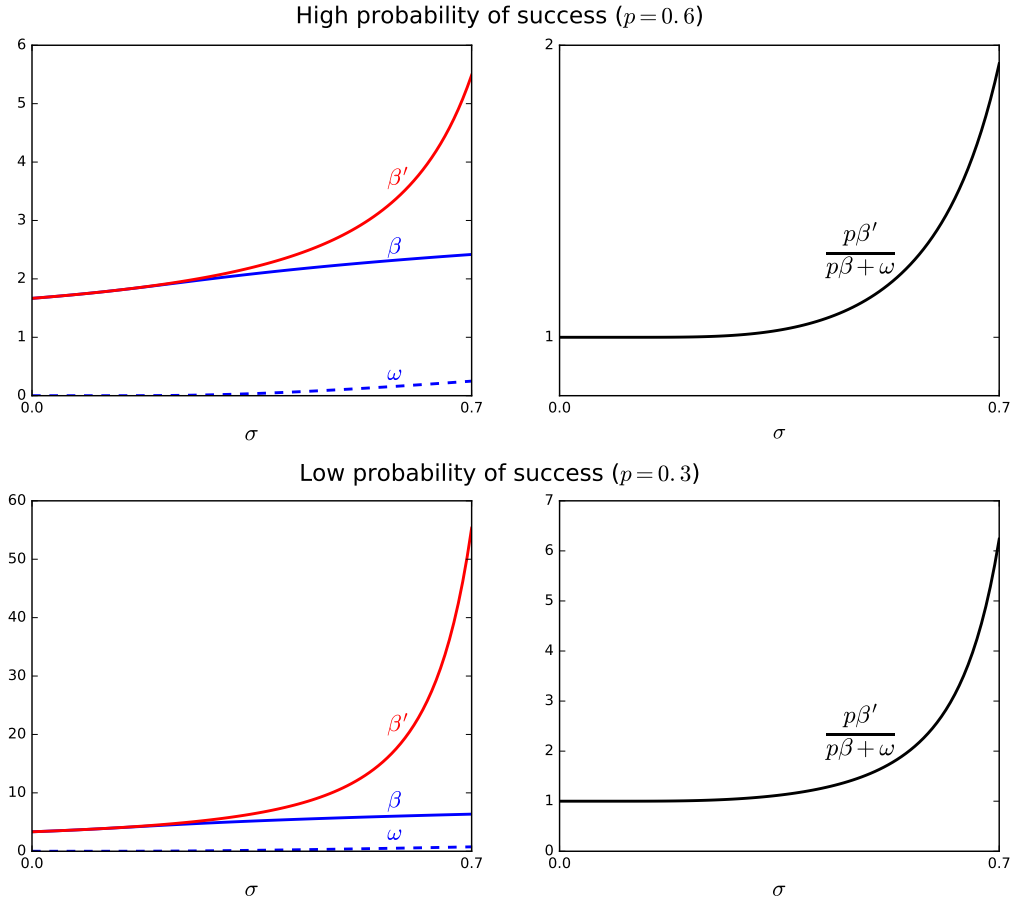
Figure 3.1 shows the optimal and ‘no insurance’ contracts for different values of relative risk aversion σ and probability of high output realization p . The parameter σ is kept below 1, because only then ‘no insurance’ contract can motivate effort. The rows correspond to different probabilities of the high output realization under effort. The left column presents the coefficients ω and β of the optimal contract and the coefficient β' of ‘no insurance’ contract. The right column shows the relative cost of providing incentives with ‘no insurance’ contract.

⁶Suppose that $u''' \leq 0$ at some interval $(0, \bar{c})$ with $\bar{c} > 0$. We can bound $u''(0)$ from below by $u''(\bar{c})$, which in turn allows us to bound $u'(0)$ from above by $u'(\bar{c}) - \bar{c}u''(\bar{c}) < \infty$.

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When the risk aversion is low, the minimal compensation is minuscule - the optimum is indistinguishable from ‘no insurance’ contract. As the risk aversion increases, both ω and β steadily rise. However, for ‘no insurance’ contract to provide the same incentives, β' has to grow much quicker. When σ approaches 1, β' diverges to $+\infty$, while ω and β remain finite. The relative cost of providing incentives without a guaranteed compensation is thus exploding. When the probability of high output realization is low, it is much harder to provide incentives for effort without insurance. As a result, β' and the cost gap between the two contracts increase even faster with the relative risk aversion.

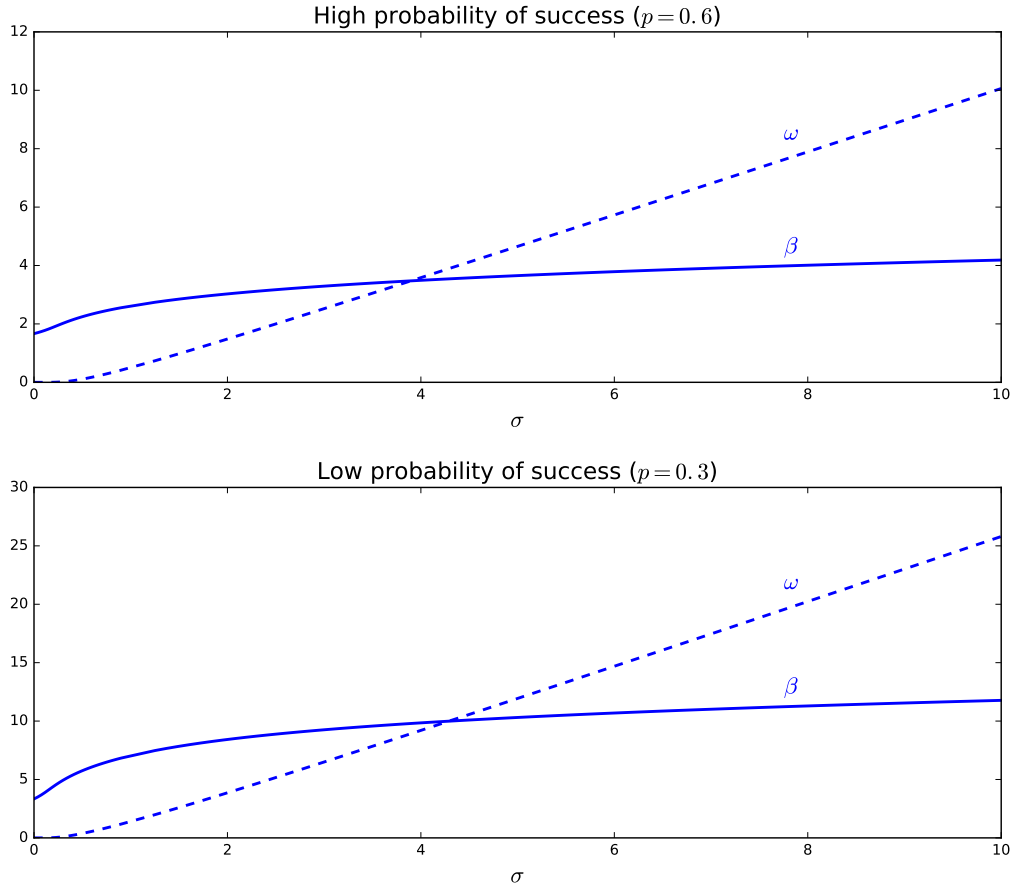
Figure 3.1: Comparison of the optimal and ‘no insurance’ contracts



It is notable that a rather small minimal compensation can lead to huge differences in the cost of incentives. The exercise is conducted for low values of the relative risk aversion, since only then

the comparison with ‘no insurance’ contract is possible. Figure 3.2 shows that for higher values of the risk aversion the guaranteed compensation ω can be substantial and exceed the value of bonus β .

Figure 3.2: The optimal contract for high levels of the risk aversion



3.5 Application to taxation

In this section I propose two interpretations of the theoretical framework studied above. In both environments there is a government that sets up an income tax with the sole aim of maximizing the tax revenue. Moreover, I assume that the utility functions of individuals feature the unbounded marginal utility at zero consumption.

Consider an entrepreneur endowed with wealth $\epsilon > 0$. The wealth can be either consumed or invested in a venture that is risky, yet profitable in expectations. This investment decision, which is unobserved by the government, affects the distribution of entrepreneur's income. Moreover, suppose that with positive probability venture fails to produce any value. In this case the government cannot tell apart the unlucky entrepreneurs from the agents that simply consumed their endowment. Nevertheless, by Proposition 3.3 providing transfers to entrepreneurs with no income improves incentives for risky investment and leads to higher tax revenue. [Albanesi \(2006\)](#) characterizes the optimal taxation of entrepreneurial income in the presence of moral hazard. However, she assumes that the cost of effort is additively separable from consumption, which by Proposition 3.1 precludes any incentive role of the positive transfer for unlucky entrepreneurs. It is natural to think that entrepreneurs' consumption is not independent of their investment choices, since the raised funds are frequently used to throw lavish startup parties.⁷

Alternatively, interpret the agent as an individual who considers going to college. The monetary cost of college ϵ becomes the sum of admission fees and foregone earnings. If both educated and uneducated workers face a possibility of zero labor productivity, which can be interpreted as chronic unemployment or disability, then by Proposition 3.3 transfers to workers with no earnings improve the incentives for education. Hence, when the return to education is sufficiently high, the sole incentive provision can justify a redistributive income tax even if the government cares only about the total tax revenue. Note that, similarly to [Badel and Huggett \(2014\)](#), I assume that the government cannot base its policies on the individual's education decision. An alternative approach, in which the government optimizes with respect to both the income tax and education subsidies, was explored by [Bovenberg and Jacobs \(2005\)](#) and [Krueger and Ludwig \(2013\)](#).

3.6 Conclusions and extensions

The relation between insurance and incentives is not necessarily monotone. Although no risk and full insurance precludes incentives, not always full risk and no insurance, i.e. paying the agent a constant fraction of output, implies the strongest incentives. I show that when effort and consumption are complements, increasing insurance by introducing an unconditional minimal pay can strengthen incentives for effort. When the cost of effort is monetary, it happens only when agents are prudent and always when the marginal utility of consumption is unbounded at zero consumption. I argue that these results are policy relevant. They highlight the efficiency role of unconditional cash transfers in encouraging a costly investment, be it an entrepreneurial activity or education.

⁷'Going too big with the launch party' is the first entry on the list of common startup mistakes in Porges, S. (2013, May 17). The 10 PR Disasters All Startups Need To Avoid. *Forbes*. Retrieved from <http://www.forbes.com>.

In the remainder of this section I discuss two possible extensions. The assumption of binary effort simplifies the analysis, since we need to consider only a single incentive constraint. Some results generalize to the case of continuous effort. I will show that the ‘if’ part of Proposition 3.3 holds also in this case, namely: $\lim_{c \rightarrow 0} u'(c) = \infty$ implies that a positive minimal is optimal. Suppose that the agent chooses the effort $e \in [0, 1]$. The effort affects the probability mass function $p_e(y)$, which is differentiable in effort at each output level. I assume that $p_e(y) \geq \underline{p} > 0$ for all $y \in Y$ and all effort levels $e \in [0, 1]$, which precludes the ‘unpleasant theorem’ of [Mirrlees \(1999\)](#). Suppose that the principal wants to implement the effort level $e^* > 0$. Then the agent’s expected utility from exerting effort e is $\sum_y p_e(y)u(w(y) + (e^* - e)\epsilon)$.⁸ Suppose that the marginal utility is unbounded at zero and that there is an output level $y \in Y$ with $w(y) = 0$. By marginally decreasing effort, the agent gains an unbounded amount in utility terms by avoiding the zero consumption, while loses at most a finite value due to the affected distribution of output. Hence, the unbounded marginal utility at zero consumption implies a positive minimal compensation.

The presented model is static. [Spear and Srivastava \(1987\)](#) express a dynamic moral hazard model with a promised-utility approach, where the agent’s compensation consists of an immediate payoff and future utility promises, which need to be fulfilled by the principal. On the one hand, the dynamic problem of the principal involves additional promise-keeping constraints which can give raise to the positive minimal pay even without the incentive justification. On the other hand, the promise-keeping constraint in the moral hazard model is expressed as equality. The principal cannot provide neither less nor more utility than promised. If the promised utility along some output path is decreasing, it’s likely that so will the minimal positive pay. [Rogerson \(1985\)](#) and [Thomas and Worrall \(1990\)](#) show that, with an additively separable disutility from effort and a utility from consumption which is unbounded below, the promised utility converges to $-\infty$ with probability 1. The investigation of the limiting behavior of the promised utility with complementarity between effort and consumption is an interesting research topic, however it is beyond the scope of this paper.

⁸The principal, besides paying the compensation w , provides the agent with resources to cover the cost of effort.

Appendix

Additional proofs

Proof of Lemma 3.1. Denote the inverse function of u with g and the inverse function of u' with h . We can express the bonus as a function of $w(0)$ with the (IC) constraint

$$w(\bar{y}) = g\left(\frac{1}{p}u(w(0) + \epsilon) - \frac{1-p}{p}u(w(0))\right). \quad (3.5)$$

By Proposition 3.3 we know that the minimal compensation is positive. We can use the interior optimality condition (3.1) with respect to $w(0)$ and $w(\bar{y})$ to obtain

$$w(\bar{y}) = h\left(u'(w(0)) - \frac{1}{1-p}u'(w(0) + \epsilon)\right). \quad (3.6)$$

Combining both equations and dividing by $w(0)$, we get

$$\left(\frac{1}{p}(1 + \epsilon/w(0))^{1-\sigma} - \frac{1-p}{p}\right)^{\frac{1}{1-\sigma}} = \left(1 - \frac{1}{1-p}(1 + \epsilon/w(0))^{-\sigma}\right)^{-\frac{1}{\sigma}}. \quad (3.7)$$

The equation above is affected by ϵ or $w(0)$ only through the ratio $\epsilon/w(0)$. It means that if we perturb ϵ and adjust $w(0)$ to keep the ratio constant, the equation will be satisfied. Hence, there exists $\omega(\sigma, p)$ such that $w(0) = \omega(\sigma, p)\epsilon$. Now take (3.5), subtract $w(0)$ from both sides and plug $\omega(\sigma, p)$ on the right-hand side to get

$$w(\bar{y}) - w(0) = \left[\left(\frac{1}{p}(1 + \omega(\sigma, p)^{-1})^{1-\sigma} - \frac{1-p}{p}\right)^{\frac{1}{1-\sigma}} - 1\right] \omega(\sigma, p)\epsilon, \quad (3.8)$$

which defines the term $\beta(\sigma, p)$. □

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